Math 631 Measure and Integration, Fall 2023

Time:	WF 8:30-9:45 am	Instructor: Xiuyuan Cheng
Classroom:	Gross Hall 318	Email: xiuyuan.cheng@duke.edu

Synopsis:

This course develops measure theory and Lebesgue integration. This theory is foundational for much of analysis and in particular for the mathematical theory of probability. Besides Lebesgue measure and integration, we include a brief introduction to classical Banach space and abstract measure theory, and the formulation of measure-theoretical probability theory (law of large numbers, central limit theorem, conditional expectation, and martingales). Time permitting, we will go to further topics in Fourier analysis, including Hilbert spaces, orthonormal bases and Fourier transforms. The course is aimed at beginning graduate students in mathematics, but other interested students are welcome if they have the appropriate background.

Textbook:

Royden and Fitzpatrick, *Real Analysis*, Chapters 1-7, selected content in Chapters 9-20.

References:

Reed and Simon, *Methods of Mathematical Physics I: Functional Analysis*, Chapters 1-2.
Stein and Shakarchi, *Real Analysis: Measure Theory, Integration & Hilbert Spaces*.

Prerequisite:

Mathematics 531, 532 or equivalent. Students should have a working knowledge of the concepts from undergraduate analysis and should be used to writing proofs in analysis. Specifically, students should be familiar with these concepts: algebra of sets and images and inverse images of functions, sup and inf of sets of real numbers, countable and uncountable sets, completeness of the real numbers, Cauchy sequences, metric spaces, open and closed sets, compact sets (as defined by coverings by open sets), the Bolzano-Weierstrass Theorem, uniform continuity as distinguished from continuity, the relation between continuous functions and compact or connected sets, pointwise convergence and uniform convergence of sequences of functions, convergence of power series, the Riemann integral as defined as the limit of upper and lower sums, and the proofs of standard theorems of calculus.

Grading Policy:

Evaluation will be based on homework assignments (30%), two midterms (30%), and one final exam (40%). All exams will be in-class. Late hand-in will not be counted towards grades except for special excuses (following the academic rules of the school). If a reschedule of an exam is needed, please contact the instructor as early as possible. Lecture notes will be distributed as the class goes on. Homework will be assigned weekly.