Assignment 7
(Due October 23)

Reading: (from Reed) §4.1

Problems: §3.2: #7, 8, 9, 11
§3.3: #1, 2, 3, 6, 8

Additional Problems: 1. Prove that if a function is continuous on the open interval \((a, b)\) and bounded on \([a, b]\), then it is Riemann integrable on \([a, b]\). (\textit{Suggestion:} Let \(f\) be the function. Prove that for any \(\epsilon > 0\) there is a partition \(P\) of \([a, b]\) such that

\[ U_P(f) - L_P(f) \leq \epsilon \]

Use your experience with #2, §3.3 to control the potential problems near the endpoints.) Conclude that the function \(f\), defined on \([0, 1]\) by \(f(x) = \sin(1/x)\) for \(x \in (0, 1]\) and \(f(0) = 7\), is Riemann integrable on \([0, 1]\).

2. Prove that if \(f(x)\) and \(g(x)\) are Riemann integrable on \([a, b]\), then \(f(x) + g(x)\) is Riemann integrable on \([a, b]\). (\textit{Suggestion:} Prove first that for any partition \(P = \{x_0, x_1, \ldots, x_N\}\) of \([a, b]\),

\[ M_i(f + g) \leq M_i(f) + M_i(g) \quad \text{and} \quad m_i(f) + m_i(g) \leq m_i(f + g) \]

where as usual \(M_i\) and \(m_i\) denote the sup and inf on \([x_{i-1}, x_i]\).)