Assignment 3
(Due September 18)

Reading: (from Reed) §2.4, 2.5

Problems: §1.3: #3(c) (the sets need not be disjoint!), 8 (ditto)
§2.1: #2b, 3b, 4b, 9b

Additional Problems: 1. Since \( a_n = \frac{n(n+1)}{2}, n = 1, 2, \ldots \), is an increasing sequence of positive integers, every integer is in one and only one of the subsets \( I_n \) of \( \mathbb{N} \) where

\[
I_1 = \{1\}, \quad I_n = (a_{n-1}, a_n] \cap \mathbb{N} = \{n(n-1)/2 + 1, \ldots, n(n+1)/2\}, n \geq 2
\]

Let \( f : \mathbb{N} \to \mathbb{N} \times \mathbb{N} \) be the function defined by the following rule. Let \( \ell \in I_n \), say \( \ell = n(n-1)/2 + k \), where \( k \in \{1, \ldots, n\} \). Then define \( f(\ell) = (n-k+1, k) \). Prove that \( f \) is bijective.

2. The logical format of the statement “The sequence \( \{a_n\} \) converges” is \( (\exists a \in \mathbb{R})(\forall \epsilon > 0)(\exists N \in \mathbb{R})(\forall n)(n \geq N \implies |a_n - a| \leq \epsilon) \). Write the statement “The sequence \( \{a_n\} \) does not converge” in logical format.

3. Show that the sequence \( \{r^n\} \) does not converge if \( r \leq -1 \). (Suggested steps: Begin with a careful statement of what you are trying to prove. Take \( \epsilon = 1/4 \) in this statement. If \( |a - r^n| > 1/4 \), (what is \( n \)?) you’re good. If not, notice that \( |r^n - r^{n+1}| \geq 1 \) (prove it!), then use the inequality of #10, §1.1 to conclude that \( |a - r^{n+1}| > 1/4. \)