Assignment 2
( Due Thursday, September 12, 2006)

Reading: §1.3; A.3; Problem 38, §1.3
Problems: §1.2: #1, 2, 4, 6, 8, 16b
§A.3: #1, 3, 4

Additional Problems: 1. Prove:
   a. If \( a, b \in \mathbb{Z} \) and \( a \mid b \), then \( |a| \leq |b| \).
   b. If \( a, b, c \in \mathbb{Z} \), \( c \mid b \) and \( b \mid a \), then \( c \mid a \).

2. Let \( a \in \mathbb{Z} \) and let \( D(a) \) be the set of divisors of \( a \). Let \( a = bq + r \), where \( b, q, r \in \mathbb{Z} \). Prove that \( D(a) \cap D(b) = D(r) \cap D(b) \).

3. Let \( a, b \in \mathbb{Z} \) and let
   \[ L(a, b) = \{ ma + nb \mid m, n \in \mathbb{Z} \}, \]
   be the set of linear combinations of \( a \) and \( b \). Let \( a = bq + r \), where \( q, r \in \mathbb{Z} \). Prove that
   \[ L(a, b) = L(r, b). \]

4. Let \( a, b, d \in \mathbb{Z} \) and suppose that \( d \mid a \) and \( d \mid b \). Prove that \( d \mid (a, b) \), where \( (a, b) \) denotes the greatest common divisor of \( a \) and \( b \). (This shows that the definition of greatest common divisor of \( a \) and \( b \) given on p.13 of the text is the same as that given in class: \( \max\{D(a) \cap D(b)\} \).)

5. Challenge problem: §1.2, #19 (You may assume \( (a, b) = 1 \).)