Math 121 February 25, 2005

Assignment 6
(Due Thursday, March 3, 2005)

Reading: §4.1

Problems: §3.2: #2(a), 4(a,c), 6(b,c,g), 7(b), 11, 15
§3.3: #2(a,c), 3(c,e), 4(a), 8

For the **Additional Problems**, assume the following. Let F be a subfield of a field K. Let $\alpha \in K$ and suppose $f(x) \in F[x]$ satisfies the coditions,

(1) $f(\alpha) = 0$
(2) $f(x)$ is irreducible in $F[x]$.

1. Let

$$F[\alpha] = \{p(\alpha) |\, p(x) \in F[x]\}$$

Prove that $F[\alpha]$ is a subfield of K.

2. Prove that each element $\beta \in F[\alpha]$ can be expressed in one and only one way,

$$\beta = a_0 + a_1\alpha + \cdots + a_{d-1}\alpha^{d-1}$$

where $a_0, a_1, \ldots, a_{d-1} \in \mathbb{Q}$ and $d = \deg f(x)$.