Math 121 January 20, 2005

Assignment 2
(Due Thursday, January 27, 2005)

Reading: §1.3, A.3, Problem 38, §1.3

Problems: §1.2: #1, 2, 3, 4, 6, 16b
§A.3: #1, 3, 4

Additional Problems: 1. Prove:

a. If a, $b \in \mathbb{Z}$ and $a|b$, then $|a| \leq |b|$.
b. If a, b, $c \in \mathbb{Z}$, $c|b$ and $b|a$, then $c|a$.

2. Let $a \in \mathbb{Z}$ and let $D(a)$ be the set of divisors of a. Let $a = bq + r$, where b, q, $r \in \mathbb{Z}$. Prove that $D(a) \cap D(b) = D(r) \cap D(b)$.

3. Let a, $b \in \mathbb{Z}$ and let

$$L(a, b) = \{ma + nb \,|\, m,\, n \in \mathbb{Z}\},$$

the set of linear combinations of a and b. Let $a = bq + r$, where q, $r \in \mathbb{Z}$. Prove that

$$L(a, b) = L(r, b).$$

4. Challenge problem: §1.2, #19 (You may asume $(a, b) = 1$.)