1

3.19 on page 182 of Mendelson. 3.28, 3.29 on page 186 of Mendelson. 3.32, 3.33, 3.34 on page 192 of Mendelson.

Note that in the problem that has $ne$ in it the $ne$ should be $n!e$.

**Hint for the problem that says a certain function is nonrecursive.** Let $f_1, f_2, \ldots, f_n, \ldots$ be an enumeration of the recursive functions of one argument. Let $g(n) = f_n(n) + 1$ for $n \in \mathbb{N}$. Show that $g$ is not recursive.

2

On page 189 in Hodel do all of Problem 4 and Parts 1, 2, 3 of Problem 5.

Note that for Problem 5 you are referred to $\exists$ ELIM on page 186. I find that material extremely confusing. Below is a solution of Part 4 of Problem 5; if you like, mimic what I do below to do Parts 1, 2, 3 of Problem 5.

We want to show

$$\{\exists x \ P(x), \forall x \ (P(x) \rightarrow Q(x)), \forall x \ (P(x) \rightarrow R(x))\} \vdash \exists x \ (Q(x) \land R(x))$$

We begin by showing

$$\{P(x), \forall x \ (P(x) \rightarrow Q(x)), \forall x \ (P(x) \rightarrow R(x))\} \vdash (Q(x) \land R(x))$$

Indeed,

$$P(x)$$
$$\forall x \ (P(x) \rightarrow Q(x))$$
$$\forall x \ (P(x) \rightarrow R(x))$$
$$(P(x) \rightarrow Q(x))$$
$$(P(x) \rightarrow R(x))$$
$$Q(x)$$
$$R(x)$$
$$(Q(x) \land R(x))$$

you supply the reasons. Now use the Deduction Theorem (being careful to note that this is allowed in this case; that’s because the reasons above, which you supplied, are what they are) to obtain

$$\{\forall x \ (P(x) \rightarrow Q(x)), \forall x \ (P(x) \rightarrow R(x))\} \vdash (P(x) \rightarrow (Q(x) \land R(x)))$$

Finally, apply one of the $\exists$ rules and MP to complete the argument that (*) holds.

3

Show that addition and multiplication are strongly representable. You will need Proposition 1.1 from Expressibility and Representability.
Let $\text{Stmt}$ be the logical function of one variable such that $\text{Stmt}(y) = 1$ if and only if $y$ is the code of a statement. Let

$$\text{Bound}$$

be the logical function of three variables such that $\text{Bound}(y, i, j) = 1$ if and only if $y$ is the code of a statement $s$, $1 \leq i \leq |s|$, $1 \leq j \leq |s|$, $s_i = x_j$ and this occurrence of $x_j$ in $s$ is bound. Using the fact that $\text{Stmt}$ is recursive describe how you would show that $\text{Bound}$ is recursive.