Final Problem Set. Due Thursday, December 11 at 5pm.

1

3.19 on page 182 of Mendelson. 3.28, 3.29 on page 186 of Mendelson. 3.32, 3.33, 3.34 on page 192 of Mendelson.

Note that in the problem that has ne in it the ne should be $n!e$.

Hint for the problem that says a certain function is nonrecursive. Let $f_1, f_2, \ldots, f_n, \ldots$ be an enumeration of the recursive functions of one argument. Let $g(n) = f_n(n) + 1$ for $n \in \mathbb{N}$. Show that g is not recursive.

2

On page 189 in Hodel do all of Problem 4 and Parts 1,2,3 of Problem 5.

Note that for Problem 5 you are referred to ∃ ELIM on page 186. I find that material extremely confusing. Below is a solution of Part 4 of Problem 5; if you like, mimic what I do below to do Parts 1,2,3 of Problem 5.

We want to show

$$(*) \qquad \{\exists x\, P(x), \forall x\, (P(x) \to Q(x)), \forall x\, (P(x) \to R(x))\} \vdash \exists x\, (Q(x) \wedge R(x))$$

We begin by showing

$$\{P(x), \forall x\, (P(x) \to Q(x)), \forall x\, (P(x) \to R(x))\} \vdash (Q(x) \wedge R(x))$$

Indeed,

$$\begin{array}{l} P(x) \\ \forall x\, (P(x) \to Q(x)) \\ \forall x\, (P(x) \to R(x)) \\ (P(x) \to Q(x)) \\ (P(x) \to R(x)) \\ Q(x) \\ R(x) \\ (Q(x) \wedge R(x)) \end{array}$$

you supply the reasons. Now use the Deduction Theorem (being careful to note that this is allowed in this case; that's because the reasons above, which you supplied, are what they are) to obtain

$$\{\forall x\, (P(x) \to Q(x)), \forall x\, (P(x) \to R(x))\} \vdash (P(x) \to (Q(x) \wedge R(x)))$$

Finally, apply one of the ∃ rules and MP to complete the argument that (*) holds.

3

Show that addition and multiplication are strongly representable. You will need Proposition 1.1 from Expressibility and Representability.

4

Let Stmt be the logical function of one variable such that Stmt$(y) = 1$ if and only if y is the code of a statement. Let

$$\text{Bound}$$

be the logical function of three variables such that Bound$(y, i, j) = 1$ if and only if y is the code of a statemtent s, $1 \leq i \leq |s|$, $1 \leq j \leq |s|$,

$$s_i = x_j$$

and this occurrence of x_j in s is bound. Using the fact that Stmt is recursive describe how you would show that Bound is recursive.