1. Disjunctive normal form.

Suppose \( P \) is a nonempty finite set of propositional variables. For each subset \( I \) of \( P \) let \( p_I = \left( \bigwedge_{p \in I} p \right) \land \left( \bigwedge_{p \in P \setminus I} \sim p \right) \).

Evidently,

\[
\text{t}_{p_I}(J) = \begin{cases} 1 & \text{if } J = I, \\ 0 & \text{if } J \neq I. \end{cases}
\]

If \( \mathcal{I} \) is a family of subsets of \( P \) we say the statement \( \bigvee_{I \in \mathcal{I}} p_I \) is in disjunctive normal form.

**Proposition 1.1.** Suppose \( T : 2^P \to \{0, 1\}, \quad \mathcal{I} = \{ I : I \subset P \text{ and } T(I) = 1 \} \) and \( B = \bigvee_{I \in \mathcal{I}} p_I. \)

Then \( \text{t}_B = T. \)

**Proof.** This follows immediately from (1). \( \square \)

**Corollary 1.1.** Suppose \( A \) is a statement and \( P \) is the set of its propositional variables. Let \( \mathcal{I} = \{ I : I \subset P \text{ and } \text{t}_A(I) = 1 \}. \)

Then \( A \leftrightarrow \bigvee_{I \in \mathcal{I}} p_I \) is a tautology.

**Proof.** Once you figure out what it says it’s obvious. \( \square \)

In logic jargon this Theorem says that any statement is tautologically equivalent to a statement in disjunctive normal form.