1. Disjunctive normal form.

Suppose

$$P$$

is a nonempty finite set of propositional variables. For each subset I of P let

$$p_I = \left(\bigwedge_{p \in I} p \right) \wedge \left(\bigwedge_{p \in P \sim I} \sim p \right).$$

Evidently,

$$\mathbf{t}_{p_I}(J) = \begin{cases} 1 & \text{if } J = I, \\ 0 & \text{if } J \neq I. \end{cases} \tag{1}$$

If $\mathcal{I}$ is a family of subsets of P we say the statement

$$\bigvee_{I \in \mathcal{I}} p_I$$

is in disjunctive normal form.

Proposition 1.1. Suppose

$$T : 2^P \to \{0, 1\}, \quad \mathcal{I} = \{I : I \subset P \text{ and } T(I) = 1\}$$

and

$$B = \bigvee_{I \in \mathcal{I}} p_I.$$

Then

$$\mathbf{t}_B = T.$$

Proof. This follows immediately from (1). □

Corollary 1.1. Suppose A is a statement and P is the set of its propositional variables. Let

$$\mathcal{I} = \{I : I \subset P \text{ and } \mathbf{t}_A(I) = 1\}.$$

Then

$$A \leftrightarrow \bigvee_{I \in \mathcal{I}} p_I \quad \text{is a tautology.}$$

Proof. Once you figure out what it says it's obvious. □

In logic jargon this Theorem says that any statement is tautologically equivalent to a statement in disjunctive normal form.