1. 5 pts. Suppose $X$ is a random variable with variance 5. Compute Var$(3X + 9)$.

2. 5 pts. Suppose $X$ and $Y$ are independent variables with expectations 3 and 4, respectively. Compute $E(XY)$.

3. 10 pts. How many 12 letter strings can be made with 3 A’s, 4 B’s and 5 C’s?

4. 10 pts. A fair six sided die is thrown 3 times. Describe a sample space for this experiment and compute the probability that the sum of the three numbers is five.

5. 20 pts. Suppose $X$ is a random variable such that

$$P(X = 1) = \frac{1}{8}, \quad P(X = 3) = \frac{1}{2}, \quad P(X = 5) = \frac{3}{8}.$$ 

Three balls are drawn from and urn containing $X$ black balls and three white balls. Let $B$ be the event that two of the three balls are black. Compute $P(X = 5 | B)$.

6. 15 pts. Suppose $X_1, X_2, \ldots, X_n, \ldots$ is a sequence of independent identically distributed random variables such that

$$E(X_i) = 4 \quad \text{and} \quad \text{Var}(X_i) = 4, \; i = 1, 2, \ldots.$$ 

Let

$$S = \sum_{i=1}^{100} X_i.$$ 

Use the Central Limit Theorem to approximate

$$P(S > 425).$$

(If you do it correctly the arithmetic is simple.)

7. 15 pts. Let

$$Q = \{(x, y) \in \mathbb{R}^2 : x \text{ and } y \text{ are integers, } x \geq 0, \; y \geq 0 \text{ and } x + y \leq 2\}.$$ 

(I suggest you draw a picture of $Q$.)
There are random variables $X$ and $Y$ such that
\[ p_{X,Y}(x, y) = \begin{cases} \frac{x + y}{8} & \text{if } (x, y) \in Q, \\ 0 & \text{else}. \end{cases} \]

Calculate the mean and variance of $X + Y$ and determine if $X$ and $Y$ are independent.

\textbf{8. 20 pts.} Suppose $A, B, C, D$ are independent events. Compute
\[ P((A \cup B) \cap (C \cup D)) \quad \text{and} \quad P((A \sim B) \cup (C \sim D)) \]

in terms of $P(A), P(B), P(C), P(D)$. 