A very useful formula.

Let \((S, \mathcal{E}, P)\) be a probability space.

**Proposition.** Suppose

(i) \(F_1, \ldots, F_n\) are disjoint events with positive probability and

\[ P(F_i) = P(F_j) \quad \text{whenever } i, j \in \{1, \ldots, n\}; \]

(ii) \(E\) is an event and

\[ P(E|F_i) = P(E|F_j) \quad \text{whenever } i, j \in \{1, \ldots, n\}. \]

Then

\[ P(E|\bigcup_{i=1}^n F_i) = P(E|F_j) \quad \text{whenever } j \in \{1, \ldots, n\}. \]

**Proof.** Suppose \(j \in \{1, \ldots, n\}\). We have

\[
P(E \cap (\bigcup_{i=1}^n F_i)) = P(\bigcup_{i=1}^n (E \cap F_i)) = \sum_{i=1}^n P(E \cap F_i) = \sum_{i=1}^n P(E|F_i)P(F_i) = nP(E|F_j)P(F_j)
\]

and

\[
P(\bigcup_{i=1}^n F_i) = \sum_{i=1}^n P(F_i) = nP(F_j)
\]

from which the desired equation immediately follows. \(\square\)

**Example. Page 66, n. 2c.** In bridge, what is the probability East gets 3 spades given that North and South have 8 spades between them.

**Solution.** Let \(E\) be the event that East gets 3 spades and let \(F\) be the event that North and South have 8 spades between them. Let

\[
S = \left( \begin{array}{c}
\text{Cards} \\
13 \\
13 \\
13 \\
13
\end{array} \right).
\]

Evidently

\[ P(\{s\}) = P(\{t\}) \quad \text{and} \quad P(E|\{s\}) = P(E|\{t\}) \quad \text{whenever } s \in F. \]

Thus, by the preceding Proposition,

\[ P(E|F) = P(E|\{s\}) \quad \text{whenever } s \in F. \]

Let \(s \in F\). I hope it’s clear that

\[ P(E|\{s\}) = \frac{\binom{5}{3}\binom{21}{10}}{\binom{26}{13}} \approx .339. \]