How many ways can you put m things in n boxes?

The answer is

$$\binom{n-1+m}{m}.$$

In order to see this, let

$$S(n,m)$$

be the set of n-tuples $\alpha = (\alpha_1, \ldots, \alpha_n)$ of nonnegative integers such that

$$\sum_{i=1}^{n} \alpha_i = m.$$

I hope it's clear that our assertion is equivalent to the following

Theorem.

$$|S(n,m)| = \binom{n-1+m}{m}.$$

Proof. Induct on $n+m$. If $n+m=1$ it's obvious so let us assume that $n+m>1$. Note that

$$|\{\alpha \in S(n,m) : \alpha_n = 0\}| = |S(n-1,m)|$$

and that

$$|\{\alpha \in S(n,m) : \alpha_n > 0\}| = |S(n,m-1)|;$$

The first of these assertions is obvious and the second follows by associating to each $\alpha \in \{\alpha \in S(n,m) : \alpha_n > 0\}$ the element

$$(\alpha_1, \ldots, \alpha_n - 1) \in S(n, m-1).$$

Thus

$$|S(n,m)| = |S(n-1,m)| + |S(n,m-1)| = \binom{n-2+m}{m} + \binom{n-1+m-1}{m-1} = \binom{n-1+m}{m}.$$

The second of these equations is the inductive step and the third is something we have already shown.

Remark. The form of the answer suggests yet another proof.