How many ways can you put \( m \) things in \( n \) boxes?

The answer is

\[
\binom{n - 1 + m}{m}.
\]

In order to see this, let

\[ S(n, m) \]

be the set of \( n \)-tuples \( \alpha = (\alpha_1, \ldots, \alpha_n) \) of nonnegative integers such that

\[
\sum_{i=1}^{n} \alpha_i = m.
\]

I hope it’s clear that our assertion is equivalent to the following

**Theorem.**

\[
|S(n, m)| = \binom{n - 1 + m}{m}.
\]

**Proof.** Induct on \( n + m \). If \( n + m = 1 \) it’s obvious so let us assume that \( n + m > 1 \). Note that

\[
|\{\alpha \in S(n, m) : \alpha_n = 0\}| = |S(n - 1, m)|
\]

and that

\[
|\{\alpha \in S(n, m) : \alpha_n > 0\}| = |S(n, m - 1)|;
\]

The first of these assertions is obvious and the second follows by associating to each \( \alpha \in \{\alpha \in S(n, m) : \alpha_n > 0\} \) the element

\[
(\alpha_1, \ldots, \alpha_n - 1) \in S(n, m - 1).
\]

Thus

\[
|S(n, m)| = |S(n - 1, m)| + |S(n, m - 1)| = \binom{n - 2 + m}{m} + \binom{n - 1 + m - 1}{m - 1} = \binom{n - 1 + m}{m}.
\]

The second of these equations is the inductive step and the third is something we have already shown.

**Remark.** The form of the answer suggests yet another proof.