

**A second final exam take home problem.**

Consider the vibrating string problem with  $L = 1$ :

(PDE) 
$$\frac{\partial^2 u}{\partial t^2} = 4 \frac{\partial^2 u}{\partial x^2};$$

(BC) 
$$u(0, t) = 0 = u(L, t);$$

(IC) 
$$u(x, 0) = f(x), \quad \frac{\partial u}{\partial t}(x, 0) = g(x).$$

Solve it in the following three cases:

$$f(x) = \sin \pi x, \quad g(x) = 0, \quad 0 < x < L.$$

$$f(x) = 0, \quad g(x) = 2 \sin 2\pi x; \quad < x < L.$$

$$f(x) = 4 \sin 3\pi x, \quad g(x) = 6 \sin 5\pi x; \quad 0 < x < L.$$