A take home final exam problem.

Introduction. We have already considered the following. Let $f \in \mathcal{P}$ be such that

$$f(x) = x \quad \text{if } -\pi \le x < \pi.$$

Then $\hat{f}(0) = 0$ and

$$\hat{f}(n) = \frac{2\pi}{in}(-1)^{n+1} \quad \text{if } n \neq 0.$$

(This is different from the first version in which there were mistakes!)

Note that if n is a positive integer and $x \in \mathbf{R}$ then

$$\frac{1}{2\pi}\left(\frac{2\pi}{in}(-1)^{n+1}e^{inx} + \frac{2\pi}{-in}(-1)^{-n+1}e^{-inx}\right) = \frac{2(-1)^{n+1}}{n}\sin nx$$

so that, as $\hat{f}(0) = 0$ we obtain

$$P_N f(x) = \sum_{|n|\le N} \hat{f}(n)E_n(x) = \frac{1}{2\pi}\sum_{0<|n|\le N} \frac{2\pi}{in}(-1)^{n+1}e^{inx} = 2\sum_{n=1}^{N}\frac{(-1)^{n+1}}{n}\sin nx.$$

Now I would like to have a picture of how $P_N f$ differs from f, say from -3π to 3π which is three periods. Here are Maple commands to this for $N = 10$.

This sets up displaying and plotting on my computer.

```
> plotsetup(x11):
> with(plots):
```

This calculates $P_{10}f$ and creates an associated plot structure. '$**$' below is exponentiation.

```
> f:=0:
> for n from 1 to 10 do:  g:=f:  f:=g+2*(-1)**(n+1)*sin(n*x)/n:  od:
> p:=plot(f,x=-3*Pi..3*Pi):
```

This sets up and creates plots of f on each of the three periods $(-3\pi, -\pi)$, $(-\pi, \pi)$, $(\pi, 3\pi)$. I'm going to plot it over three periods so you can see what happens near the jumps. I'll do it with dashed lines.

```
> L1:=[[-3*Pi,-Pi],[-Pi,Pi]];
> L2:=[[-Pi,-Pi],[Pi,Pi]];
> L3:=[[Pi,-Pi],[3*Pi,Pi]];
> p1:=plot(L1,linestyle=DASH):
> p2:=plot(L2,linestyle=DASH):
> p3:=plot(L3,linestyle=DASH):
```

Finally, the whole works is displayed together. This is when you get the picture. On my computer I can save the picture and then print it.

```
> display(p,p1,p2,p3);
```

Now here is what I want you to do.

I want you to do the *same* thing only this time $f \in \mathcal{P}$ is such that

$$f(x) = \begin{cases} x + \pi & \text{if } -\pi \le x < 0, \\ \pi - x & \text{if } 0 \le x < \pi. \end{cases}$$

f is a so-called sawtooth function. Be careful, though. This time $\hat{f}(0)$ is *not* zero and where one had sin's above you're going to get cos's. To compute the Fourier coefficients you can (i) do it by hand (not recommended); (ii) use the Jump Formula (you'll have to use it *twice*!); or (iii) use Maple.