The computer assignment.

Consider the IVP

$$y'(x) = 1 - y(x)^2, \quad y(0) = 0.$$

In class we showed that the unique solution to this IVP was defined for all x and satisfied $|y(x)| < 1$ for all x.

Let

$$\mathbf{E}(h, N)$$

be the result of applying Euler's method for N steps with a stepsize of h and let

$$\mathbf{errE}(h, N) = y(Nh) - \mathbf{E}(h, N).$$

Let

$$\mathbf{ME}(h, N)$$

be the result of applying the modified Euler method for N steps with a stepsize of h and let

$$\mathbf{errME}(h, N) = y(Nh) - \mathbf{ME}(h, N).$$

It is a fact that

$$\mathbf{errE}(h, [1/h]) \sim C_{\mathbf{E}} h$$

for some constant $C_{\mathbf{E}}$ and that

$$\mathbf{errME}(h, [1/h]) \sim C_{\mathbf{ME}} h^2$$

for some constant $C_{\mathbf{ME}}$.

(1) Determine the exact solution $y(x)$ of the IVP. It would be nice if you used Maple or some other symbolic calculator to do this.

Let

$$h_1 = .1, \quad h_2 = .01, \quad h_3 = .001$$

and let

$$N_1 = 10, \quad N_2 = 100, \quad N_3 = 1000.$$

(2) What do expect $\ln \mathbf{errE}(h_i, N_i)$ and $\ln \mathbf{errME}(h_i, N_i)$, $i = 1, 2, 3$, to be?

(3) Plot $\ln \mathbf{errE}(h_i, N_i)$ and $\ln \mathbf{errME}(h_i, N_i)$, $i = 1, 2, 3$.

IF YOU USE `eulersteps`, ETC., BE SURE TO ENTER h AS A DECIMAL!