The computer assignment.

Consider the IVP
\[ y'(x) = 1 - y(x)^2, \quad y(0) = 0. \]
In class we showed that the unique solution to this IVP was defined for all \( x \) and satisfied \( |y(x)| < 1 \) for all \( x \).

Let
\[ E(h, N) \]
be the result of applying Euler’s method for \( N \) steps with a stepsize of \( h \) and let
\[ \text{err} E(h, N) = y(Nh) - E(h, N). \]

Let
\[ ME(h, N) \]
be the result of applying the modified Euler method for \( N \) steps with a stepsize of \( h \) and let
\[ \text{err} ME(h, N) = y(Nh) - ME(h, N). \]

It is a fact that
\[ \text{err} E(h, [1/h]) \sim C_E h \]
for some constant \( C_E \) and that
\[ \text{err} ME(h, [1/h]) \sim C_{ME} h^2 \]
for some constant \( C_{ME} \).

(1) Determine the exact solution \( y(x) \) of the IVP. It would be nice if you used Maple or some other symbolic calculator to do this.

Let
\[ h_1 = .1, \quad h_2 = .01, \quad h_3 = .001 \]
and let
\[ N_1 = 10, \quad N_2 = 100, \quad N_3 = 1000. \]

(2) What do expect \( \ln \text{err} E(h_1, N_i) \) and \( \ln \text{err} ME(h_1, N_i), i = 1, 2, 3, \) to be?

(3) Plot \( \ln \text{err} E(h_i, N_i) \) and \( \ln \text{err} ME(h_i, N_i), i = 1, 2, 3. \)

**IF YOU USE eulersteps, ETC., BE SURE TO ENTER \( h \) AS A DECIMAL!**