Hilbert’s axioms for (two dimensional) neutral geometry.

We spell these out below. It will take a while. There will be several groups of axioms: the incidence axioms; the betweenness axioms; the continuity axiom; and the congruence axioms.

The Incidence Axioms. There are a set whose members we call points and a family of sets of points whose members we call lines such that

(I1) if $a$ and $b$ are distinct points there is one and only one line $l(a, b)$, the line determined by $a$ and $b$, such that \{a, b\} $\subseteq$ $l(a, b)$;

(I2) any line contains at least two points.

If $p$ is a point and $L$ is a line we say $p$ lies on $L$ if $p \in L$.

Theorem. If $L$ and $M$ are lines and $L$ intersects $M$ then either $L = M$ or $L \cap M$ contains exactly one point.

Proof. This follows directly from (I1). □

If $L$ and $M$ are lines and $L$ does not intersect $M$ we say $L$ and $M$ are parallel.

Definition. Suppose $S$ is a set of points. We say $S$ is collinear if $S$ is a subset of some line. We say $S$ is noncollinear if $S$ is not collinear.

Note that a subset of a collinear set is collinear and that a superset of a noncollinear set is noncollinear.

(I3) There is a noncollinear set of points.

An obvious consequence of (I3) is the following.

Theorem. Suppose $L$ is a line. Then there is a point which does not lie on $L$. 
