Homework One, due September 1

1. Show that the incidence axioms hold in the standard model for Euclidean geometry.

2. Show that

\[ ((p \rightarrow q) \land (q \rightarrow r)) \rightarrow (p \rightarrow r) \]

is a tautology.

3. Let \( n \) be a positive integer \( n \). All matrices we talk about below have either real or complex entries. A set \( G \) of matrices is called a **group** if

(i) \( I \in G \);

(ii) If \( A \in G \) and \( B \in G \) then \( AB \in G \);

(iii) If \( A \in G \) then \( A \) is invertible and \( A^{-1} \in G \).

Let \( \text{GL}(n) \) be the set of invertible \( n \times n \) matrices and note that it is a group. (It’s called the **general linear group**. Let \( \text{A}(n) \) be the set of \( (n+1) \times (n+1) \) matrices having the block form

\[
\begin{bmatrix}
A & a \\
0 & 1
\end{bmatrix}
\]

where \( A \in \text{GL}(n) \); \( a \) is an \( n \times 1 \) matrix; \( 0 \) is the \( 1 \times n \) matrix of all zeroes; and \( 1 \) is the \( 1 \times 1 \) matrix whose entry is 1. Show that \( \text{A}(n) \) is a group. (It’s called the **affine group**). Exhibit a formula for the inverse in this group.