Math 577: Mathematical Modeling

A graduate course on mathematical formulation, simplification, and solution of applied problems

"*Everything should be made as simple as possible, but no simpler.*" ≈ Albert Einstein

MATH 577-01 MATHEMATICAL MODELING [5178]
Spring 2024, WF 1:25-2:40 pm, Room 235 Physics Bldg
http://www.math.duke.edu/∼witelski/577

This course will present an introduction to analytical methods and mathematical models for problems in the applied sciences and engineering. Problems from areas like mechanical systems, control theory, bio-chemical reactions, and heat transfer will be formulated as idealized mathematical models. Governing equations will be derived from first principles in geometry, physics, and the calculus of variations. Applied math techniques such as nondimensionalization, scaling parameter-dependence, perturbation analysis, and self-similar solutions will then be introduced to simplify the models and yield insight on the solutions of the original full problems.

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Textbook: *Applied Mathematics* (4th Ed) by J. D. Logan, Wiley (2013).

Web resources: **Canvas MATH 577.01.Sp24** page – Primary source for notes and homeworks
Back-up webpage: http://www.math.duke.edu/∼witelski/577

Background: Prerequisites for this course are undergraduate courses in differential equations (at the level of Math 353 or 356 or equivalents), basic physics (mechanics), and multi-variable calculus (Math 212). Background material will be concisely reviewed when needed.

Course Grade:[1] Based on the two mid-term exams (50%), and weekly homeworks (50%).
The course will run until the last week of classes. Last lecture: **Wed April 24**.
Auditing students are welcome to attend the course on an un-graded basis.
You are strongly encouraged to study and interact with your classmates!

Homework:[2] Assignments to be submitted using **Gradescope**. No unexcused late assignments will be accepted. **You are encouraged to discuss the homework problems with your classmates, but your final submission must be the product of your independent work.** Assignments can be expected to require several hours of work – plan ahead. Email Tom for hints on questions rather than spending too much time being stuck.

Office hours: Regular weekly times to be determined, or by email for appointment requests.

Reference books: Logan is the only required textbook for this course. Supplementary notes will be distributed when needed. Some other books that may be helpful for additional explanations or examples:

- *Introduction to the Foundations of Applied Mathematics* by M. H. Holmes
- *Mathematical Physiology* by J. Keener and J. Sneyd
- *Practical Applied Mathematics* by S. Howison
- *Mathematical Models in the Applied Sciences* by A. C. Fowler
- *Mathematics Applied to Deterministic Problems in the Natural Sciences* by C. C. Lin and L. A. Segel

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[1] Prior approval or an official excuse letter are required for accommodations/adjustments to any assigned work.

[2] The Duke Community Standard will be assumed in full effect throughout this course "I have adhered to the Duke Community Standard in completing this assignment."

Course Outline

Part 0: **General introduction to the mathematical modeling of problems** 1.1.1

Part 1: **Dimensional analysis and scaling**: parameter-dependent problems/solutions Chapter 1

- Dimensional analysis and the Buckingham Π-theorem 1.1
- Dimensional Scaling 1.2
- Scaling symmetries and self-similar solutions of differential equations
- Nondimensionalization and dimensionless parameters

Part 2: **Perturbation methods** Chapter 3

- Introduction to asymptotic analysis 3.1.4
- Regular perturbation problems 3.1
- Singular perturbation problems 3.2
- Boundary layer analysis 3.3

Part 3: **Dynamical systems and reaction rate kinetics**

- Describing vibrations of weakly nonlinear oscillators 3.1.2
  - The Poincare-Lindstedt expansion 3.1.3
  - The method of multiple scales
- Fast/slow dynamical systems 3.4
  - Review of phase plane analysis 2.1-2.4
  - Chemical reaction kinetics 2.5
  - The quasi-steady state assumption and the Michaelis-Menten model 3.4

Part 4: **Optimization and control problems**

- Calculus of Variations Chapter 4
  - The Euler-Lagrange equation and extensions 4.3-4.4
  - Hamiltonian dynamics 4.5
  - Lagrange multipliers for constrained optimization 4.6
- Optimal control of dynamical systems

Part 5: **Wave behavior and PDE transport models** Chapters 6, 7, 8

- Kinematics and moving coordinates for deforming materials 6.2, 8.1, 8.4
- Conservation laws for continuum mechanics 6.2, 8.1
- Waves 7.1–7.4

Part 6: **Reduced models for PDE problems and applications**

- Dimensional reduction: Asymptotics for problems in slender domains
- Reduced order models: The method of moments
- Turing Instability and Taylor Dispersion