Math 575: Mathematical Fluid Dynamics

A graduate course on the mathematical study of fluid mechanics

MATH 575-01 MATHEMATICAL FLUID DYNAM [4837]
Spring 2022, WF 3:30-4:45 pm, Room 205 Physics Bldg
http://www.math.duke.edu/∼witelski/575

This course gives an overview of fluid dynamics from a mathematical viewpoint, and introduces areas of active research in fluid dynamics. This course is targeted for graduate students in mathematics, physics, and engineering. Topics covered will include: mathematical formulations of free surface flows, the shallow water equations, low Reynolds number flows, Stokes flow, lubrication theory, boundary layers, classic laminar flows, potential flows, hyperbolic partial differential equations, and stability analysis.

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Textbook:[1] *Elementary Fluid Dynamics* by D. J. Acheson, Oxford University Press, 1990.

Web resources: **Sakai MATH.575.01.Sp22** page – Resources folders (primary source for notes)
Webpage http://www.math.duke.edu/∼witelski/575

Background: Undergraduate background in physics (basic mechanics), vector calculus, linear algebra, ordinary differential equations and linear partial differential equations is needed. Other material will be reviewed when needed in the course.

Course Grade: To be based on the weekly homework assignments.
The course will run until the last week of classes, Wed April 20.
Auditing students are welcome to attend the course on an un-graded basis.
You are encouraged to study with your classmates.

Homework:[2] No unexcused late assignments will be accepted. **You are encouraged to discuss the homework problems with your classmates, but your final written submission must be the product of your own independent work.** Weekly assignments can be expected to require several hours of work. Email Tom with questions rather than spending too much time being stuck.

Office hours: (Schedule to be announced), Room 295 Physics Building, or by appointment(send email)

Reference books: Some are available at Perkins library or as e-books (*):

- *Fundamental Mechanics of Fluids* by I. G. Currie (*)
- *An Introduction to Theoretical Fluid Mechanics* by S. Childress (*)
- *Fluid Mechanics* by P. K. Kundu and I. M. Cohen (*)
- *A Mathematical Introduction to Fluid Mechanics* by A. J. Chorin and J. E. Marsden (*)
- *Theoretical Fluid Dynamics* by A. Feldmeier
- *Incompressible Flow* by R. L. Panton (*)
- *Theoretical Hydrodynamics* by L. M. Milne-Thomson

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[1] Acheson's book gives a very good overall coverage of fluid dynamics, but we will not follow it in his given order. You may find some of the other reference books very helpful for various sections of the course.

[2] **Standard disclaimer: The pledge to obey the details of the Duke Community Standard for conduct and academic work will be assumed in full effect throughout this course:** ***"I have adhered to the Duke Community Standard in completing this assignment."***

Course Outline (v 0.1)[3]

(I) The mathematical formulation of fluid dynamics	Sections
1. **Kinematics**: The geometry of fluid flows	
Flow visualization	1.2
Rate-of-strain, shear, and rotation	6.4, 6.2
Eulerian/Lagrangian descriptions	1.2
2. **Dynamics**: Conservation law equations	
The Reynolds transport theorem	6.3
Conservation of mass: the continuity equation	1.3
Conservation of momentum: Cauchy's equations	6.3
Euler's equations	1.3
Constitutive laws for fluid properties: the stress tensor	6.3, 6.4
The Navier-Stokes equations	6.4, 2.2
Initial and boundary conditions	2.2, 3.2
Bernoulli's equation	2.2, 3.2
The vorticity equation	1.4, 1.5, 5.1
(II) Solving the equations of fluid dynamics	
1. **Exact solutions**	
Irrotational flows and potential flows	4.2
Complex variables for 2D flows (airfoils, point vortices, ...)	4.3–4.8
Viscous laminar flows	2.3, 2.4
2. **Asymptotic solutions**: overview	
Nondimensional parameters in fluid dynamics	2.2
Introduction to perturbation methods	
3. **Low Reynolds number flows**	
Stokes' equations	7.1, 7.4, 7.5
Lubrication theory: the Reynolds equation	7.6–7.10
Stokes' paradox	7.2
4. **High Reynolds number flows**	
Boundary layer theory	8.1–8.3
5. **Water waves**	
The shallow water equations	3.9, 3.10
Surface waves on deep water	3.2
Linearized analysis	3.1, 3.3–3.5
The KdV equation	3.11
(III) Introduction to flow instabilities	
Linear stability analysis	
Interfacial instabilities: Rayleigh-Taylor, Kelvin-Helmholtz	
Critical parameters and other instabilities	9.2, 9.4

[3] may get revised/updated