MATH 553: Asymptotic Analysis and Perturbation Methods

\[ u + \frac{x}{4} \frac{du}{dx} = \frac{d^4 u}{dx^4} \quad \xrightarrow{\text{standard}} \quad u(x) = \int_C s^3 e^{-s^4} e^{xs} ds \]

\[ \xrightarrow{\text{asymptotics}} \quad u(x) \sim \sqrt{\frac{2\pi}{3}} \left( \frac{x}{4} \right)^{2/3} e^{-\frac{3}{2}(\frac{x}{4})^{4/3}} \cos \left( \sqrt{\frac{27}{4}} \left( \frac{x}{4} \right)^{4/3} + \frac{19\pi}{12} \right) \]

Course outline: Asymptotic analysis and perturbation methods provide powerful techniques in applied mathematics for obtaining simple analytical forms to reliably approximate solutions to complicated problems in a range of different mathematical settings. The course will cover asymptotic expansions, solution of nonlinear algebraic equations, regular and singular perturbations problems, matrix eigenvalue problems, asymptotics of integrals - Fourier and Laplace transforms, and solutions of differential equations - singular points, WKB theory, multiple-scale analysis, boundary layers, and matched asymptotic expansions.

Prerequisites: Background in ordinary differential equations (Math 353, 356 or higher), undergraduate background in multi-variable calculus (line integrals or contour integrals from complex variables).


Web: http://www.math.duke.edu/~witelski/553
Email: witelski@math.duke.edu