Asymptotic analysis and perturbation methods provide powerful techniques for obtaining simple analytical forms for approximate solutions (that become exact in appropriate limits) for a wide range of mathematical problems. This course will cover material on asymptotic expansions, solutions of nonlinear algebraic equations, regular and singular perturbations, perturbations of matrix eigenvalue problems, asymptotics of integrals (Fourier and Laplace transforms), and solutions of differential equations (WKB theory, eigenvalue problems, multiple-scale analysis, boundary layers, and matched asymptotic expansions).

Background: Prerequisites include undergraduate-level background in ordinary differential equations (Math 353, 356 or higher) and basic background in multi-variable calculus (line integrals or contour integrals from complex variables). Material on complex variables will be concisely reviewed when needed in the course.

Course Grade: Based on weekly problem sets (70%) and Final Exam (30%): Saturday November 21, 2020.

Homework: Assignments to be submitted using Gradescope.com. No unexcused late assignments will be accepted without prior approval. You are encouraged to discuss the homework problems with your classmates, but your final written submission must be the product of your own independent work.

Office hours: (Schedule to be announced) Online: Zoom Meetings, or by appointment (send email).

Reference books: B&O is the only required textbook for this course, supplementary notes will be made available when needed. Some other books that may be helpful for additional explanations or examples:

- *Perturbation Methods* by E. J. Hinch
- *Introduction to Perturbation Methods* by M. H. Holmes
- *Applied Asymptotic Analysis* by P. D. Miller
- *Introduction to Perturbation Techniques* by A. H. Nayfeh

1Paraphrased from a quote from the Internet...
2The pledge to obey the details of the Duke Community Standard for conduct and academic work will be assumed in full effect throughout this course: “I have adhered to the Duke Community Standard in completing this assignment.” If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment.

COVID-19: Students in this course are expected to abide by the commitments they made in signing the Duke Compact to protect the health and safety of their fellow students, faculty, staff, families and neighbors. First time, minor violations of COVID-19 conduct expectations will be met with appropriate educational responses. However, anyone who fails to comply with the expectations of the Duke Compact more than once, or who flagrantly commits a serious violation that creates a health or safety risk to others in the Duke community, will be subject to more significant consequences, beginning with loss of the privilege to attend courses in-person and/or loss of access to campus, and moving up to suspension or expulsion.
(1) **Introduction**
   Asymptotic approximations of functions
   Perturbation parameters, limits, and asymptotic relations: $O, o, \sim, \gg$
   Asymptotic series expansions: Taylor, Laurent, Frobenius, General Asymptotic
   Perturbation methods for solving algebraic equations
   Solution of perturbed matrix eigenvalue problems
   Regular and Singular perturbation problems

(2) **Asymptotics for integrals**
   Elementary methods
   Watson’s lemma for Laplace-type integrals
   Stationary phase for Fourier-type integrals
   Steepest descent and saddle points in the complex plane

(3) **Asymptotics for Ordinary Differential Equations I**
   Local expansions of solutions: regular and irregular singular points
   Irregular singular points at infinity
   The WKBJ method

(4) **Asymptotics for Ordinary Differential Equations II**
   Matched asymptotic expansions
   Singular perturbations: boundary layers
   The method of multiple scales: nonlinear oscillators