PDE’s in polar coordinates

0. Reading from Haberman: Section 2.5.2, Sections 7.7.1-4, 7.7.9.

1. Haberman, page 82, problem 2.5.3b.
   Note that the domain is $a \leq r \leq \infty$ and do not forget the $n = 0$ special case!

2. Haberman, page 83, problem 2.5.6b.

3. Haberman, page 83, problem 2.5.8c.
   Determine the solvability condition and set-up (but do not solve) the algebra for the four sets of constant coefficients in the eigenfunction expansion.
   Note: While I’d normally recommend using superposition to split up problems with multiple inhomogeneous boundary conditions, this yields trouble when you need to deal with a FAT solvability condition. I recommend NOT splitting this problem!

   This problem is about solving the wave equation, $u_{tt} = c^2 \nabla^2 u$, where $c$ is a positive constant (the wavespeed), in the given domain with $u = 0$ homogeneous Dirichlet boundary conditions. The natural frequencies of vibration are given in terms of the eigenvalues by $\omega = c\sqrt{\lambda}$.
   Determine the equation for $\lambda$ by seeking a nontrivial solution in separation of variables form, $u = \frac{f(r)g(\theta)h(t)}{g(\theta)}$ (sometimes called a “normal mode”), that satisfies all of the boundary conditions.
   Hint: A $n \times n$ system of homogeneous linear equations has nontrivial solutions if the determinant of the coefficient matrix is zero.

   This problem is on the disk, $0 \leq r \leq a$ and $0 \leq \theta \leq 2\pi$, with homogeneous Dirichlet boundary conditions, $u(a, \theta) = 0$.
   (a) First, obtain the general solution for general initial conditions as a double summation (with four sets of constant coefficients).
   (b) Then apply the given initial conditions to reduce to a single summation (with a single set of coeffs).

Test 2 (date to be determined...) Material covered: Green’s functions for ODE BVP’s (Haberman 9.3), separation of variables and eigenfunction expansions for PDE’s [heat, wave, Laplace, Poisson, Helmholtz, etc] (2.3, 2.4, 8.2–8.4, 8.6), multi-dimensional problems (2.5, 7.2–7.10), Lectures 10-25, and Homeworks 5-8.
Like Test 1, you can use the basic-math summary sheet and you can make one sheet of notes (no books or calculators).
Since PDE separation-of-variables problems can be long, you will be asked to work out only specific parts of such full problems; follow instructions carefully and provide solutions in the forms specified in the questions.

Other Graduate Applied Mathematics Courses (Spring 2021)

- Math 557: Introduction to PDE – other approaches for studying linear and nonlinear PDEs (waves, Green’s fnns, etc) (semi-theoretical)
- Math 563: Applied Computational Analysis – numerical methods for ODEs (computational/applied)
- Math 577: Mathematical Modeling – formulation and simplifying several areas of physical problems (diffusion, chemical reactions, dynamics, perturbation methods) via math approaches (ODE/PDE) (applied)