Math 551: Applied Partial Differential Eqns and Complex Vars

A graduate course on analytical methods for linear differential equations

MATH 551 APP PART DIFF EQU & COMPX VAR [4836]
Fall 2021, MWF 5:15-6:05 pm, Room 259 Physics Building
http://www.math.duke.edu/~witelski/551

This course covers classic applied math methods for solving problems in linear partial differential equations based on generalized Fourier series and orthogonal eigenfunction expansions. Theory covered includes linear operators and adjoint problems, Sturm-Liouville eigenvalue problems and related topics: integral equations, solutions via Green’s functions, complex variables for contour integrals, and solutions via integral representations (Fourier and Laplace integral transforms).

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Web resources: Sakai MATH.551.01.F21: Resources folders, Ed discussion tool (main Q&A discussion)
http://www.math.duke.edu/~witelski/551 – back-up webpage
Course Grade: Based on TWO midterm tests (40%), Final Exam (30%), and weekly homeworks (30%).
Students are welcome to audit the course on an un-graded basis.

Tests:¹ There will be two in-class midterm tests and a cumulative Final Exam on Sun Dec 12, 2021.
Lectures will run until the last day of fall semester: Fri Dec 3, 2021.
No calculators/software may be used on tests. You are encouraged to study with your classmates.
Homework:² Assignments to be submitted using Gradescope.com. No unexcused late assignments will be accepted without prior approval. You are encouraged to discuss the homework problems with your classmates, but your final written submission must be the product of your own independent work. Weekly assignments can be expected to require several hours (2, 3, 4, · · · 6?) of work – start early/plan ahead! Go to office hours, post on Sakai/Ed, work with others, and email Tom for hints/questions rather than spending too much time being stuck.

Office hours: Zoom Meetings/In-person. Weekly schedule to be announced, or by appointment²
Prerequisites: undergraduate courses in linear algebra (like Math 216, 218 or 221) and ordinary differential equations (Math 353 or 356). Background will be concisely reviewed when needed.
Reference books: Haberman is the only required textbook for this course, supplementary notes will be made available when needed. Some other books that may be helpful for additional explanations or examples:

- Complex variables and applications by R. V. Churchill and J. W. Brown
- Fourier series and boundary value problems by R. V. Churchill and J. W. Brown
- Applied Mathematics (3rd Ed) by J. D. Logan
- A first course in partial differential eqns with complex variables and transforms by H. F. Weinberger

¹Prior approval or an official excuse letter are required to be excused from a test.
²The pledge to obey the details of the Duke Community Standard for conduct and academic work will be assumed in full effect throughout this course: “I have adhered to the Duke Community Standard in completing this assignment.” If a student is found responsible through the Office of Student Conduct for academic dishonesty on a graded item in this course, the student will receive a score of zero for that assignment.
COVID-19: Students in this course are expected to abide by the commitments they made in signing the Duke Compact to protect the health and safety of their fellow students, faculty, staff, families and neighbors...
Course Outline

(I) Basic Linear Theory and Orthogonal Expansions

Review of Linear Algebra
- Matrix eigenvalue problems and IVP for vector ODE systems

Review of Fourier Series
- Orthogonal eigenfunction expansions, properties, and examples

(II) ODE boundary value problems

Eigenvalue problems for ODEs
- Linear differential operators and adjoint problems
- Explicitly solvable equations
- Sturm-Liouville theory for self-adjoint problems
- Singular Sturm-Liouville problems
- Inhomogeneous problems: solution via eigenfunction expansions
- The Fredholm Alternative Theorem
- Fredholm integral equations

Green’s functions for ODEs
- Integral representations of solutions of BVPs
- Distribution theory: Dirac delta function and Heaviside step function

(III) PDE problems

Review of Separation of Variables

Eigenfunction expansions
- Problems for the heat equation
- Problems for the wave equation
- Problems for the Poisson equation

Problems in 2D and 3D: multi-dimensional expansions
- Problems for the Helmholtz equation
- Bessel functions and problems in cylindrical coordinates
- Legendre polynomials and problems in spherical coordinates

Green’s functions for PDEs
- The Poisson equation and boundary integrals

(IV) Integral transform methods for ODEs and PDEs

Complex Variables
- Theory of analytic functions of a complex variable
- Contour integrals and Cauchy’s theorem
- Evaluation of integrals via the Residue theorem

Fourier Transforms

Laplace Transforms

Overview

The main goals of the course are in constructing analytical formulas for solutions for partial differential equations (PDE) problems. Applications of PDEs can include propagation of electromagnetic waves in Maxwell’s equations, pressure waves in acoustics, diffusion of temperature in the heat equation, convective and diffusive mass transport, mechanical stress determined by Laplace’s equation, dynamics of elastic plates and beams, wavefunctions in Schrödinger’s equation and many other problems.

More generally, the overall process and the techniques used in constructing the solutions are more important than the formula to solve any one specific problem. As presented in Math 551, Linear theory forms a big part of the scientific language, framework, and terminology that is shared by mathematics, the applied sciences, and engineering. Linear theory is used for describing behaviors in wide classes of linear and nonlinear problems. Math 551 connects to many applied areas (stability theory, dynamical systems, bifurcation theory, control theory, numerical methods) as well as more theoretical ones (functional analysis).