1. (21 pts) Evaluate the trigonometric integral
\[
\int_0^{2\pi} \frac{\cos \theta}{5 + 4 \cos \theta} d\theta.
\]

2. (28 pts) Calculate the principal value of the integral
\[
\int_{-\infty}^{\infty} \frac{e^{i3x}}{x(4 + x^2)} \, dx.
\]

Draw and justify the use of your integration contours.

3. (28 pts) Evaluate the following complex integral on the contour \( C \) given by the line \( z = 5 + iy \) for \( y : -\infty \to \infty \),
\[
\frac{1}{2\pi i} \int_C \frac{10e^z}{(z + 3)(z^2 + 1)} \, dz.
\]

(a) (11 pts) Justify closing the contour using a large semi-circle of the form \( z = 5 + Re^{i\theta} \) for \( R \to \infty \) (and determine the range of \( \theta \)).

(b) (11 pts) Obtain the value of the integral.

(c) (6 pts) Simplify the solution to show that it is real-valued.

4. (23 pts) Evaluate the following complex integral on the closed contour \( C \) given by the circle \( |z| = \sqrt{2} \) traced counterclockwise,
\[
\oint_C \frac{\sin(\pi z)}{z^2(z - 1) \left(z - \frac{1}{2}\right)} \, dz.
\]

Have a good Thanksgiving break.