MATH 234 QUIZ 8 10 NOV 2016, TA: JIUYA WANG

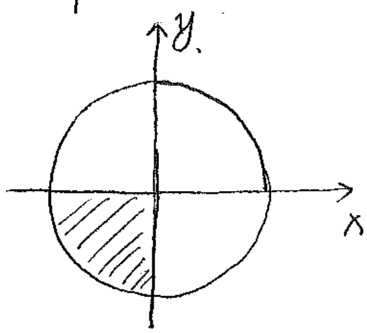
Name:

Notice:
1. Please box your final answer.
2. Please stop writing when time is up.

Problem 1 (10 points):
Compute the volume under the graph of $z = xy$ above the domain

$$D = \{(x, y) : x \le 0, y \le 0, x^2 + y^2 \le 4\}$$

Projection:



Height: $xy - 0$

$$\int_{\pi}^{\frac{3\pi}{2}} \int_0^2 r\cos\theta\sin\theta\, r \cdot dr\, d\theta$$

$$= \int_{\pi}^{\frac{3\pi}{2}} \cos\theta \cdot \sin\theta \cdot \frac{r^4}{4}\Big|_0^2 \, d\theta$$

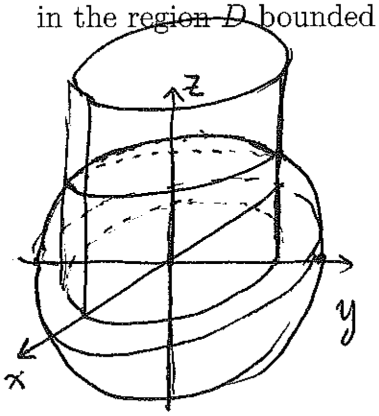
$u = \sin\theta$, $du = \cos\theta\, d\theta$

$$= 4 \cdot \int_0^{-1} u\, du = 2$$

Problem 2 (10 points):
Compute the integral:

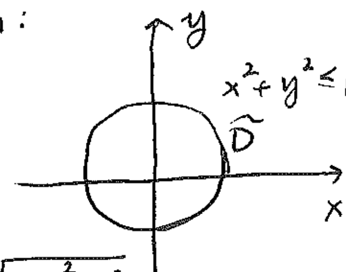
$$\iiint_D z dV$$

in the ~~region D~~ bounded by $x^2 + y^2 = 1$, $x^2 + y^2 + z^2 = 2$ and $z = 0$.



cylinder intersect with sphere:

Projection:



$$\iint_{\tilde{D}} \int_0^{\sqrt{2-x^2-y^2}} z\, dz\, dy\, dx$$

$$= \iint_{\tilde{D}} \frac{1}{2} \cdot (2 - x^2 - y^2)\, dy\, dx$$

$$= \int_0^{2\pi}\int_0^1 \frac{1}{2} \cdot (2 - r^2) \cdot r\, dr\, d\theta$$

$$= \int_0^{2\pi} \left(\frac{1}{2} - \frac{1}{8}\right) d\theta$$

$$= \frac{3}{4}\pi$$