MATH 234 QUIZ 4 6 OCT 2016, TA: JIUYA WANG

Name:

Notice:
1. Please box your final answer.
2. Please stop writing when time is up.

Problem 1 (10 points):
Given the following surface:

$$x^2 + 2y^2 + z^2 = 4$$

1. What is the tangent plane at $(1,1,1)$?
2. And what is the normal vector of this plane?

1. The surface is the level set of $f(x,y,z) = x^2 + 2y^2 + z^2 - 4$ at 0.
So. $\nabla f = \begin{pmatrix} 2x \\ 4y \\ 2z \end{pmatrix}$ $\nabla f \big|_{(1,1,1)} = \begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$ is the normal vector.
The tangent plane is $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} x-1 \\ y-1 \\ z-1 \end{pmatrix} = 0$ i.e. $x + 2y + z = 4$

Problem 2 (10 points):
Consider the level sets of the function

$$f(x,y) = x^2 + 4y^2$$

at level 4,

1. Find the points on the level set where the gradient is parallel to the vector $\vec{a} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$

2. At each point you find out in part 1, in which direction f increases fastest? in which direction f decreases the fastest? in which direction the function remains the same?

1. $\nabla f = \begin{pmatrix} 2x \\ 8y \end{pmatrix}$ $\nabla f = k \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ $\Rightarrow$ $x = \frac{k}{2}$, $y = \frac{k}{8}$

$(\frac{k}{2})^2 + 4 \cdot (\frac{k}{8})^2 = 4$ $\Rightarrow$ $k = \pm \frac{8}{\sqrt{5}}$

So there're 2 points: $P_1: (\frac{4}{\sqrt{5}}, \frac{1}{\sqrt{5}})$ and $P_2: (-\frac{4}{\sqrt{5}}, -\frac{1}{\sqrt{5}})$
corresponding to $k_1 = \frac{8}{\sqrt{5}}$ and $k_2 = -\frac{8}{\sqrt{5}}$

2. At P_1, $k_1 = \frac{8}{\sqrt{5}}$ $\nabla f = \frac{8}{\sqrt{5}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ in ∇f direction f increases fastest, in $-\nabla f = -\frac{8}{\sqrt{5}} \cdot \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ direction, f decreases the fastest.

the direction perpenticular to ∇f is $\frac{8}{\sqrt{5}} \cdot \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ or $\frac{8}{\sqrt{5}} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$

f remains the same.

For P_2: $k_2 = -\frac{8}{\sqrt{5}}$, in $\nabla f = -\frac{8}{\sqrt{5}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ direction, f increases fastest.

in $-\nabla f = \frac{8}{\sqrt{5}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ direction f decreases fastest.

in direction perpenticular to ∇f, i.e. $\frac{8}{\sqrt{5}} \begin{pmatrix} -1 \\ 1 \end{pmatrix}$ or $\frac{8}{\sqrt{5}} \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, f remains the same.