MATH 234 QUIZ 2 22 SEPT 2016, TA: JIUYA WANG

Name:

Notice:
1. Please box your final answer.
2. Please stop writing when time is up.

Problem 1 (5 points):
Compute the length of one full turn of the following helix:

$$\vec{x}(t) = \begin{pmatrix} R\cos \pi t \\ R\sin \pi t \\ at \end{pmatrix}$$

$$\vec{x}'(t) = \begin{pmatrix} -\pi R \sin \pi t \\ \pi R \cos \pi t \\ a \end{pmatrix} \qquad \| \vec{x}'(t) \| = \sqrt{\pi^2 R^2 + a^2}$$

$$\int_0^2 \sqrt{\pi^2 R^2 + a^2}\, dt = 2 \cdot \sqrt{\pi^2 R^2 + a^2}$$

Problem 2 (5 points):
Consider the curve above, compute:

1. The tangent line at $t = 1$ and its intersection with xy-plane.
2. For any given t find the tangent line and its intersection with xy-plane $P(t)$.

1. $$\vec{x}(1) = \begin{pmatrix} -R \\ 0 \\ a \end{pmatrix} \qquad \vec{x}'(1) = \begin{pmatrix} 0 \\ -\pi R \\ a \end{pmatrix} \qquad \vec{y}(s) = \begin{pmatrix} -R \\ 0 \\ a \end{pmatrix} + s \cdot \begin{pmatrix} 0 \\ -\pi R \\ a \end{pmatrix}$$

$a + s \cdot a = 0 \Rightarrow s = -1 \Rightarrow$ intersection at $(-R, \pi R, 0)$

2. $$\vec{y}(s) = \begin{pmatrix} R\cos \pi t \\ R \sin \pi t \\ at \end{pmatrix} + s \cdot \begin{pmatrix} -\pi R \sin \pi t \\ \pi R \cos \pi t \\ a \end{pmatrix}$$

$at + s \cdot a = 0 \Rightarrow s = -t \Rightarrow$ intersection at $P(t) = (R\cos \pi t + t \cdot \pi R \sin \pi t,\ R \sin \pi t - t \cdot \pi R \cos \pi t,\ 0)$