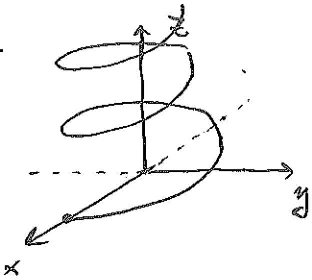
MATH 234 WORKSHEET 2 13 SEPT 2016, TA: JIUYA WANG

Problem 1 : Know the Curves

Describe the Shape of the Following Curve $\vec{v}(t)$:

1. $\begin{pmatrix} 1 \\ -1 \end{pmatrix} + t \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ line
2. $\begin{pmatrix} t \\ t^2 \end{pmatrix}$ parabola
3. $\begin{pmatrix} 3t \\ 4 \\ 1 \end{pmatrix} + 2 \begin{pmatrix} t+1 \\ t-1 \\ 2t \end{pmatrix}$ line
4. $\begin{pmatrix} 3\cos \pi t \\ 3 \sin \pi t \end{pmatrix}$ circle
5. $\begin{pmatrix} \cos \pi t \\ \sin \pi t \\ t \end{pmatrix}$ helix
6. $\begin{pmatrix} 1 + 3\cos \pi t \\ -2 + 3 \sin \pi t \end{pmatrix}$ circle

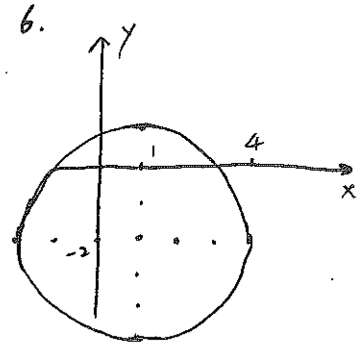
5.



$\vec{v}'(t) = \begin{pmatrix} -\pi \sin \pi t \\ \pi \cos \pi t \\ 1 \end{pmatrix}$

$\vec{v}''(t) = \begin{pmatrix} -\pi^2 \cos \pi t \\ -\pi^2 \sin \pi t \\ 0 \end{pmatrix}$

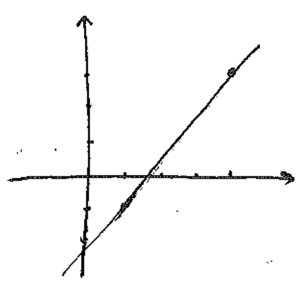
6.



$\vec{v}'(t) = \begin{pmatrix} -3\pi \sin \pi t \\ 3\pi \cos \pi t \end{pmatrix}$

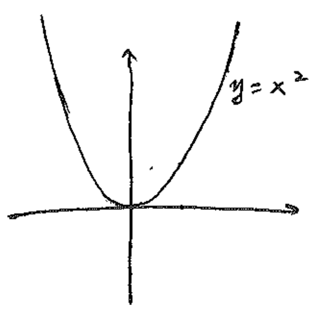
$\vec{v}''(t) = \begin{pmatrix} -3\pi^2 \cos \pi t \\ -3\pi^2 \sin \pi t \end{pmatrix}$

1.



$\vec{v}'(t) = \begin{pmatrix} 3 \\ 4 \end{pmatrix}$ $\vec{v}''(t) = \vec{0}$

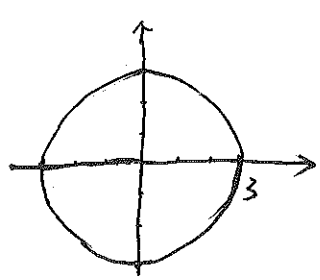
2.



$\vec{v}'(t) = \begin{pmatrix} 1 \\ 2t \end{pmatrix}$ $\vec{v}''(t) = \begin{pmatrix} 0 \\ 2 \end{pmatrix}$

3. $\vec{v}'(t) = \begin{pmatrix} 5 \\ 2 \\ 4 \end{pmatrix}$ $\vec{v}''(t) = \vec{0}$

4.



$\vec{v}'(t) = \begin{pmatrix} -3\pi \sin \pi t \\ 3\pi \cos \pi t \end{pmatrix}$

$\vec{v}''(t) = \begin{pmatrix} -3\pi^2 \cos \pi t \\ -3\pi^2 \sin \pi t \end{pmatrix}$

Problem 2: Derivation of Vector Functions
Compute $\vec{v}'(t)$ and $\vec{v}''(t)$ for the vector functions in Problem 1:

1. Compute all $\vec{v}'(t)$ for Problem 1
2. Compute all $\vec{v}''(t)$ for Problem 1
3. Compute $\frac{d}{dt}\|\vec{v}(t)\|^2$ for 3, 4, 5 in Problem 1
4. Compute $\vec{v}'(t) \times \vec{v}(t)$ for $\cancel{4}$ 5 in Problem 1

3. $\frac{d}{dt}\|\vec{v}(t)\|^2 = \frac{d}{dt}\vec{v}(t)\cdot\vec{v}(t) = \left(\frac{d}{dt}\vec{v}(t)\right)\cdot\vec{v}(t) + \vec{v}(t)\cdot\left(\frac{d}{dt}\vec{v}(t)\right)$

$= 2\cdot\vec{v}(t)\cdot\vec{v}'(t)$

for 3: $2\cdot\vec{v}(t)\cdot\vec{v}'(t) = 2\cdot\begin{pmatrix}5t+2\\2t+2\\4t+1\end{pmatrix}\cdot\begin{pmatrix}5\\2\\4\end{pmatrix} = 2(25t+10+4t+4+16t+4)$
$= 90t+36$

4: $\|\vec{v}(t)\|^2 = 9$ so $\frac{d}{dt}\|\vec{v}(t)\|^2 = 0$

5: $2\cdot\vec{v}(t)\cdot\vec{v}'(t) = 2\cdot\begin{pmatrix}\cos\pi t\\\sin\pi t\\t\end{pmatrix}\cdot\begin{pmatrix}-\pi\sin\pi t\\\pi\cos\pi t\\1\end{pmatrix} = 2t$

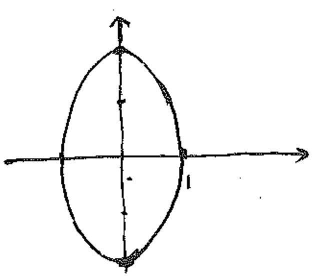
4. $\vec{v}'(t)\times\vec{v}(t) = \begin{vmatrix}\vec{i} & \vec{j} & \vec{k}\\ -\pi\cdot\sin\pi t & -\pi\cdot\cos\pi t & 1\\ \cos\pi t & \sin\pi t & t\end{vmatrix}$

$= \begin{pmatrix}-\pi t\cos\pi t - \sin\pi t\\ \cos\pi t + \pi t\sin\pi t\\ -\pi\sin^2\pi t + \pi\cos^2\pi t\end{pmatrix}$

Problem 3: Bonus Problem
Do you know these curves? What is the shape?

1. $\begin{pmatrix}\cos\pi t\\2\sin\pi t\end{pmatrix}$
2. $\begin{pmatrix}t\\\sqrt{1-t^2}\end{pmatrix}$

1. ellipse : $x^2 + (\frac{y}{2})^2 = 1$



2. half circle : $x^2+y^2=1$, $y>0$