Problem 1 (10 points):
Given the following surface:
\[ xy + yz + xz = -1 \]

1. Using implicit function theorem, if we can determine \( z = f(x, y) \) to be implicit function depending on \( x \) and \( y \), to compute \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \);
2. At which point(s) \( z \) is not an implicit function depending on \( x \) and \( y \)?
3. Find critical point(s) of function \( z = f(x, y) \) you found in part 1.

\[ \frac{\partial z}{\partial x} = -\frac{F_x}{F_z} = -\frac{y+z}{x+y} \quad \frac{\partial z}{\partial y} = -\frac{F_y}{F_z} = -\frac{x+z}{x+y} \]

2. \( F_z = 0 \Rightarrow x + y \Rightarrow -x^2 = -1 \quad x = \pm 1 \) so the points are: \((1, -1, z)\) and \((-1, 1, z)\)

3. \( \frac{\partial z}{\partial x} = 0 \Rightarrow \frac{y+z}{x+y} \Rightarrow x = z = -1 \) so. \(-x^2 = -1 \quad x = \pm 1 \).

\[ \frac{\partial z}{\partial y} = 0 \Rightarrow x + z = 0 \]

the critical points are: \((1, 1, -1)\) and \((-1, -1, 1)\)

Problem 2 (10 points):
Given
\[ f(x, y) = e^{x+y^2} \]
and let
\[ x(u, v) = u^2 - v^2, \quad y(u, v) = 2uv \]
consider the function \( g(u, v) = f(x(u, v), y(u, v)) \), compute \( \frac{\partial g}{\partial u} \) and \( \frac{\partial g}{\partial v} \).

\[ \frac{\partial g}{\partial u} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial u} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial u} = e^{x+y^2} \cdot 2u + e^{x+y^2} \cdot 2v \]
\[ \frac{\partial g}{\partial v} = \frac{\partial f}{\partial x} \cdot \frac{\partial x}{\partial v} + \frac{\partial f}{\partial y} \cdot \frac{\partial y}{\partial v} = e^{x+y^2} \cdot (-2v) + e^{x+y^2} \cdot 2y \cdot 2u \]