Problem 1 (10 points):
Given the following surface:
\[ x^2 + 2y^2 + z^2 = 4 \]

1. What is the tangent plane at (1, 1, 1)?
2. And what is the normal vector of this plane?

The surface is the level set of \( f(x, y, z) = x^2 + 2y^2 + z^2 - 4 \) at \( 0 \).
So, \( \nabla f = \left( \frac{2x}{2x}, \frac{4y}{4y}, \frac{2z}{2z} \right) \)
\( \nabla f \bigg|_{(1,1,1)} = \left( \frac{2}{2}, \frac{4}{4}, \frac{2}{2} \right) = \left( \frac{2}{2}, \frac{2}{2}, \frac{2}{2} \right) \)
is the normal vector.

The tangent plane is
\[ \left( \frac{2}{2} \right) \cdot \left( \frac{x-1}{2}, \frac{y-1}{2}, \frac{z-1}{2} \right) = 0 \]
i.e. \( x - 2y + z = 4 \).

Problem 2 (10 points):
Consider the level sets of the function
\[ f(x, y) = x^2 + 4y^2 \]
at level 4,
1. Find the points on the level set where the gradient is parallel to the vector \( \vec{a} = \left( \frac{1}{1} \right) \)
2. At each point you find out in part 1, in which direction \( f \) increases fastest? in which direction \( f \) decreases the fastest? in which direction the function remains the same?

\[ \nabla f = \left( \frac{2x}{8y} \right) \]
\( \nabla f = k \cdot (1, 1) \Rightarrow \frac{x}{2} = \frac{y}{k} \)
\( \left( \frac{k}{2} \right)^2 + 4 \cdot \left( \frac{k}{8} \right)^2 = 4 \Rightarrow k = \pm \frac{8}{\sqrt{5}} \)
sO there are 2 points: \( P_1 : \left( \frac{6}{\sqrt{5}}, \frac{1}{\sqrt{5}} \right) \) and \( P_2 : \left( -\frac{4}{\sqrt{5}}, -\frac{1}{\sqrt{5}} \right) \)
 corresPonding to \( k_1 = \frac{8}{\sqrt{5}} \) and \( k_2 = -\frac{8}{\sqrt{5}} \)

2. At \( P_1 \), \( k_1 = \frac{8}{\sqrt{5}} \)
\( \nabla f = \frac{8}{\sqrt{5}} \cdot (1, 1) \)
In \( \nabla f \) direction \( f \) increaseS fastest, in \(-\nabla f = -\frac{8}{\sqrt{5}} \cdot (1, 1) \) direction \( f \) decreaseS the fastest.
The direction perpendicular to $\nabla f$ is $\frac{\partial}{\partial x^5} \left( -1 \right)$ or $\frac{\partial}{\partial x^5} \left( -1 \right)$, $f$ remains the same.

For $P_2$: $k_2 = -\frac{8}{5}$, in $\nabla f = -\frac{8}{5 \sqrt{5}} \left( -1 \right)$ direction, $f$ increases fastest.

in $-\nabla f = \frac{8}{5 \sqrt{5}} \left( 1 \right)$ direction $f$ decreases fastest.

in direction perpendicular to $\nabla f$, i.e. $\frac{8}{\sqrt{5}} \left( -1 \right)$ or $\frac{8}{\sqrt{5}} \left( -1 \right)$, $f$ remains the same.