Problem 1 (10 points):
Consider the quadratic form
\[ f(x, y) = \frac{1}{2} x^2 - xy + y^2 \]

1. Classify the quadratic form by completing the square;
2. Classify the quadratic form by \( 4AC - B^2 \);
3. Determine the zero set of this quadratic form;

1. \[
\frac{1}{2} x^2 - xy + y^2 = \frac{1}{2} \left( x^2 - 2xy + y^2 \right) = \frac{1}{2} \left( x - y \right)^2 + \frac{1}{2} y^2
\]
   positive definite

2. \( 4AC - B^2 = 4 \cdot \frac{1}{2} \cdot 1 - (-1)^2 = 1 > 0 \)
   \( A = \frac{1}{2} > 0 \)
   positive definite

3. \( x - y = 0 \) and \( y = 0 \) \Rightarrow \( x = y > 0 \)

Problem 2 (10 points):
Consider the function:
\[ f(x, y) = \sqrt{x^2 + 2xy - 3y^2} \]

What is the largest domain for this function? Draw the domain on x-y plane.

1. \( \sqrt{x^2 + 2xy - 3y^2} \) has domain \( x^2 + 2xy - 3y^2 \geq 0 \)

2. \( x^2 + 2xy - 3y^2 = (x + y)^2 - 2y^2 = (x + y + 2y)(x + y - 2y) \geq 0 \)