Participant Abstracts
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A TIME-DEPENDENT POISSON RANDOM FIELD MODEL FOR POLYMORPHISM WITHIN AND BETWEEN TWO RELATED BIOLOGICAL SPECIES

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We derive a time-dependent Poisson random field to model genetic differences within and between two related species that share a relatively recent common ancestor. We first consider a random field of Markov chains that describes the fate of a set of individual mutations. This field is then approximated by a Poisson random field from which we can make inferences about the amounts of mutation and selection that have occurred in the history of observed aligned DNA sequences.

http://faculty.unlv.edu/amei

MODERATE DEVIATIONS FOR TWO SAMPLE T-STATISTICS

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Let $X_1, \cdots, X_{n_1}$ be a random sample from a population with mean μ_1 and variance σ_1^2, and $Y_1, \cdots, Y_{n_2}$ be a random sample from another population with mean μ_2 and variance σ_2^2 independent of $\{X_i, 1 \le i \le n_1\}$. Consider the two sample t-statistic $T = \frac{\bar{X}-\bar{Y}-(\mu_1-\mu_2)}{\sqrt{s_1^2/n_1+s_2^2/n_2}}$. This paper shows that $\ln P(T \ge x) \sim -x^2/2$ for any $x := x(n_1, n_2)$ satisfying $x \to \infty$, $x = o(n_1+n_2)^{1/2}$ as $n_1, n_2 \to \infty$ provided $0 < c_1 \le n_1/n_2 \le c_2 < \infty$. If, in addition, $E|X_1|^3 < \infty$, $E|Y_1|^3 < \infty$, then $\frac{P(T \ge x)}{1-\Phi(x)} \to 1$ holds uniformly in $x \in (0, o((n_1+n_2)^{1/6}))$.

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SELF-SIMILARITY PARAMETER ESTIMATION AND REPRODUCTION PROPERTY FOR NON-GAUSSIAN HERMITE PROCESSES

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We consider the class of all the Hermite processes $(Z_t^{q,H})_{t\in[0,1]}$ of order q and with Hurst index $H \in (\frac{1}{2}, 1)$. The process $Z^{q,H}$ is H-self-similar, it has stationary increments and it exhibits long-range dependence identical to that of fractional Brownian motion (fBm). For $q = 1$, $Z^{1,H}$ is fBm, which is Gaussian; for $q = 2$, $Z^{2,H}$ is the Rosenblatt process, which lives in the second Wiener chaos; for any $q > 2$, $Z^{q,H}$ is a process in the qth Wiener chaos. We study the variations $Z^{q,H}$ for any q, by using multiple Wiener-Itô stochastic integrals and Malliavin calculus.

We prove a reproduction property for this class of processes in the sense that the terms appearing in the chaotic decomposition of their variations give rise other Hermite processes of different orders and with different Hurst parameters. We apply our results to construct a strongly consistent estimator for the self-similarity parameter H from discrete observations of $Z^{q,H}$; the asymptotics of this estimator, after appropriate normalization, are proved to be distributed like a Rosenblatt random variable (value at time 1 of a Rosenblatt process) with self-similarity parameter $1 + (2H - 1)/q$.

`http://arxiv.org/abs/0807.1208`

ASYMPTOTICALLY OPTIMAL CONTROL OF SINGLE-CLASS TIME-INHOMOGENEOUS NETWORKS

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A framework for the study of asymptotically optimal control problems involving open single-class time-varying queueing networks is introduced by means of uniform acceleration. A characterization of a family of performance measures for which fluid-optimal policies perform well asymptotically is given via a novel notion of *fluid-continuity*. Examples of fluid-continuous performance measures are given and simulations implementing proposed asymptotically optimal policies are presented. Finally, an example of a performance measure that is not fluid-continuous is given.

DEGENERATE STOCHASTIC DIFFERENTIAL EQUATIONS FOR CATALYTIC BRANCHING NETWORKS

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Uniqueness of the martingale problem corresponding to a degenerate SDE which models catalytic branching networks is proven. This work is an extension of a paper by Dawson and Perkins to arbitrary catalytic branching networks. As part of the proof estimates on the corresponding semigroup are found in terms of weighted Hölder norms for arbitrary networks, which are proven to be equivalent to the semigroup norm for this generalized setting.

`http://www.math.ubc.ca/~kliem/`

POSITIVELY AND NEGATIVELY EXCITED RANDOM WALKS ON INTEGERS, WITH BRANCHING PROCESSES

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We consider excited random walks on integers with a bounded number of i.i.d. cookies per site which may induce drifts both to the left and to the right. We extend the criteria for recurrence and transience by M. Zerner and for positivity of speed by A.-L. Basdevant and A. Singh to this case and also prove an annealed central limit theorem. The proofs are based on results from the literature concerning branching processes with migration and make use of a certain renewal structure.

http://arxiv.org/abs/0801.1924

SPACE-TIME CURRENT PROCESS FOR INDEPENDENT RANDOM WALKS IN ONE DIMENSION

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In a system made up of independent random walks, fluctuations of order $n^{1/4}$ from the hydrodynamic limit come from particle current across characteristics. We show that a two-parameter space-time particle current process converges to a two-parameter Gaussian process. These Gaussian processes also appear as the limit for the one-dimensional random average process. The final section of this paper looks at large deviations of the current process.

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ASYMPTOTIC ANALYSIS OF MULTISCALE APPROXIMATIONS TO REACTION NETWORKS

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A reaction network is a chemical system involving multiple reactions and chemical species. Stochastic models of such networks treat the system as a continuous time Markov chain on the number of molecules of each species with reactions as possible transitions of the chain. In many cases of biological interest some of the chemical species in the network are present in much greater abundance than others and reaction rate constants can vary over several orders of magnitude. We consider approaches to approximation of such models that take the multiscale nature of the system into account. Our primary example is a model of a cell's viral infection for which we apply a combination of averaging and law of large number arguments to show that the "slow" component of the model can be approximated by a deterministic equation and to

characterize the asymptotic distribution of the "fast" components. The main goal is to illustrate techniques that can be used to reduce the dimensionality of much more complex models.

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FLUID LIMITS FOR SHORTEST REMAINING PROCESSING TIME QUEUES

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We consider a single server queue with renewal arrivals and i.i.d. service times, in which the server employs the shortest remaining processing time policy. To describe the evolution of this queue, we use a measure valued process that keeps track of the residual service times of all buffered jobs. We propose a fluid model (or formal law of large numbers approximation) for this system and, under mild assumptions, prove existence and uniqueness of fluid model solutions. Furthermore, we prove a scaling limit theorem that justifies the fluid model as a first order approximation of the stochastic model. The state descriptor of the fluid model is a measure valued function whose dynamics are governed by certain inequalities in conjunction with the standard workload equation. In particular, these dynamics determine the evolution of the left edge (infimum) of the state descriptor's support, which yields conclusions about sojourn times. We characterize the evolution of this left edge as an inverse functional of the initial condition, arrival rate, and service time distribution. This characterization reveals the manner in which the growth rate of the left edge depends on the service time distribution. It is shown that the rate can vary from logarithmic to polynomial by considering various examples. In addition, it is shown that this characterization implies that critical fluid models for which the service time distribution has unbounded support converge to the empty queue as time approaches infinity, a perhaps unexpected result since the workload remains constant.

`http://www.csusm.edu/puha`

SOME OBSERVATIONS FOR MEAN-FIELD SPIN GLASS MODELS

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We obtain bounds to show that the pressure of a two-body, mean-field spin glass is a Lipschitz function of the underlying distribution of the random coupling constants, with respect to a particular semi-norm. This allows us to re-derive a result of Carmona and Hu, on the universality of the SK model, by a different proof, and to generalize this result to the Viana-Bray model. We also prove another bound, suitable when the coupling constants are not independent, which is what is necessary if one wants to consider "canonical" instead of "grand canonical" versions of the SK and Viana-Bray models. Finally, we review Viana-Bray type models, using the language of Lévy processes, which is natural in this context.

`http://front.math.ucdavis.edu/0707.0031`

ATTRACTION, BOSONS, AND CONTAGION

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The family of beta distributions, Pólya urn schemes, Bose-Einstein models, and order statistics are interrelated with contagion models and to the behavior of bosons by means of the concept of *attraction*.

STATE SPACE COLLAPSE AND DIFFUSION APPROXIMATION FOR A NETWORK OPERATING UNDER A FAIR BANDWIDTH SHARING POLICY

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We consider a connection-level model of Internet congestion control, introduced by Massoulie and Roberts, that represents the randomly varying number of flows present in a network. Here bandwidth is shared fairly amongst elastic document transfers according to a weighted alpha-fair bandwidth sharing policy introduced by Mo and Walrand (alpha in (0,infinity)). Assuming Poisson arrivals and exponentially distributed document sizes, we focus on the heavy traffic regime in which the average load placed on each resource is approximately equal to its capacity. A fluid model (or functional law of large numbers approximation) for this stochastic model was derived and analyzed in a prior work by two of the authors. Here we use the long time behavior of the solutions of this fluid model established in that prior work to derive a property called multiplicative state space collapse, which loosely speaking shows that in diffusion scale the flow count process for the stochastic model can be approximately recovered as a continuous lifting of the workload process.

Under weighted proportional fair sharing of bandwidth (alpha=1) and a mild local traffic condition, we show how multiplicative state space collapse can be combined with an invariance principle to establish a diffusion approximation for the workload process and hence to yield an approximation for the flow count process. In this case, the workload diffusion behaves like Brownian motion in the interior of a polyhedral cone and is confined to the cone by reflection at the boundary, where the direction of reflection is constant on any given boundary face. When all of the weights are equal (proportional fair sharing), this diffusion has a product form invariant distribution. If the latter is integrable, it yields the unique stationary distribution for the diffusion which has a strikingly simple interpretation in terms of independent dual random variables, one for each of the resources of the network. We are able to extend this product form result to the case where document sizes are distributed as finite mixtures of exponentials, and to models that include multi-path routing. We indicate some difficulties related to extending the diffusion approximation result to values of alpha not equal to 1.

We illustrate our approximation results for a few simple networks. In particular, for a two resource linear network, the diffusion lives in a wedge that is a strict subset of the positive quadrant. This geometrically illustrates the entrainment of resources, whereby congestion at one resource may prevent another resource from working at full capacity. For a four resource network with multi-path routing, the product form result under proportional fair sharing is expressed in terms of independent dual random variables, one for each of a set of generalized cut constraints.

www.math.ucsd.edu/~williams/bandwidth/webcongestion.pdf