

Stochastic Calculus: A Practical Introduction, R. Durrett

Updated: May 15, 2000

Errors:

(pointed out to me by Marc Yor): (6.2) on page 121 is wrong. One cannot take $H^i \in \Pi_2(B^i)$ but must consider $H^i \in \Pi_2(B^1, \dots, B^d)$. Otherwise we would not be able to represent $\int_0^t B_t^1 dB_t^2$. The flaw in our argument is in (6.3) which is false. A correct proof can be found in Section V.3 of Revuz and Yor (1991). All that is needed to patch the current argument is to consider $\phi(z_1, \dots, z_d)$ in the proof of (6.4) as a function of d complex variables.

p.253 line -5: This is $\leq (C/t)P_x(T_H \leq t)$. To fix this introduce $J = \{x : |x| \leq R - 1/2\}$ and notice that

$$\sup_{y \in \partial K} P_y(T_H \leq t) \leq \sup_{y \in \partial K} P_y(T_J \leq t) \sup_{z \in \partial J} P_z(T_H \leq t) \leq Ct^2$$

Typos:

The cover! Second = should be +

p.5, line 5: constant misspelled

p.6, line -12: (1.3) implies

p.7, line 5: (1.3) gives

p.8 line 7: $\dots B_{s_m}$

p.9 line -6: minimizing $E(Z - X)^2$ over $X \in \mathcal{F}$ (no sub s)

p.12, Proof of Theorem (2.3): See (1.5)

p.12, Exercise 2.1. \mathcal{F}_s^+

p.12, Exercise 2.2. the function $f(x) = x_i$ is not bounded so we cannot directly apply (2.1). The remedy is simple: apply the result to $f_n(x) = x_i 1(|x_i| < n)$ and let $n \rightarrow \infty$.

p.13, Exercise 2.5, line 2: $P_x(T > 1, B_1 \in K) \geq \alpha$ line 3: $P_x(T > n, B_n \in K) \geq \alpha^n$

p.13, Exercises 2.6 and 2.7: \mathcal{F}_s^+

p.13, Exercise 2.7, second line of formula: $\exp(\int_0^s$

p.21, line -8: $\mathcal{G} = \sigma(\cup_n \mathcal{G}_n)$

p.23, lines 13–16: \mathcal{A} should be the collection of sets of the form $\{(s, \omega) : s \in G_0, \omega(t_j) \in G_j, j = 1, \dots, \ell\}$, $\ell \geq 1$. Also, the definition of f_k should be $f_k(x) = 1 \wedge k \cdot \text{dist}(x, G^c)$, so $f_k \nearrow 1_G$. (Otherwise for $x \in \partial G$ we have $f_k(x) = 1 \not\rightarrow 0$.)

p.27, line 10: $\frac{1}{\pi}(t(1-t))^{-1/2}$ (no integral sign)

p.28, line -1: add a subscript n to f

p.29, line 4: (1.3) implies

p.29, (4.7): $C_s = B^1(\tau_s)$

p.35, (*): $1_{(a,b]}(s)$ (not t)

p.37, line 4: Π , not Π'

p.37, line -3: $Y_t^{T_n}$

p.39, line 12: $E(X_T | \mathcal{F}_{S_n}) = X_{T \wedge S_n}$

p.39, line 16: the optional stopping theorem is (2.3)

p.42, Exercise 2.5: we do not need X to be continuous

p.64, (6.2): we should define \hat{H}_s^T to be 0 for $s > T$ and rewrite the conclusion as

$$\hat{H}^T \cdot X = (H \cdot X)^T = H \cdot X^T = \hat{H}^T \cdot X^T$$

p.68, line 2: formula misspelled

p.69, line 5: right bracket missing on $f''(c(\dots$

p.69, definition of A_s^n : t_{i+1}^n and t_i^n are missing superscripts

p.69, (7.7): $\int_0^t G_s^n dA_s^n$, i.e., superscript n not δ

p.78 line 3: For then (6.8) and Exercise 7.1

p.91, (12.3) Given α_t a strictly positive martingale

p.97, line -7: $x_1^2 + \dots + x_d^2$

p.101, lines 6-7: $B_t \in D^c$ and $B_t \in G^c$

p.104, line 13: $\varphi(x) = -|x|$

p.104, line 14:

$$\varphi'(x) = \begin{cases} -1 & x > 0 \\ +1 & x < 0 \end{cases}$$

p.108, line 3: no $1/2$ power on right

p.108, line 14: martingale exponential of X_t

p.108, line -11: Y is the only solution

p.109, second display from bottom: $(2\pi s^3)^{-1/2}$ not $1/2$ power

p.112, second line of first display: \int_0^t

p.112, last line: Integrate $d\theta$ not dx

p.113, second display: $= P(X'_t \in A \cap B)$

p.113, line -12: the equation $\gamma(\langle X \rangle_t) = t$ fails where γ is discontinuous. To get $X_t = B_{\langle X \rangle_t}$ apply Exercise 3.8 in Chapter 2.

p.115: the heading should be Section 3.4 not 4.3

p.115, line -7: using (4.8) not (4.9)

p.153, Proof of Theorem (5.5): In (ii) instead of $\|v\|_\infty$ one has to bound $\|\bar{v}\|_\infty$, where

$$\bar{v}(y) = E_y \int_0^\tau |g(B_s)| ds.$$

Then the last two expressions in the displayed inequalities are

$$\varepsilon \|g\|_\infty + E_{x_n}(\bar{v}(B_\varepsilon); \tau > \varepsilon) \leq \varepsilon \|g\|_\infty + \|\bar{v}\|_\infty P_{x_n}(\tau > \varepsilon).$$

p.165: linear on $[-1, y]$

p.175, line -5: inside integral should have $\sqrt{t-s}$ in denominator

p.175, line -2: no dt

p.181, line -2: β_n not β

p.183, line -5: (4.1) in Chapter 3

p.185, line -8: $\int_0^{T \wedge \tau}$

p.187, line 4: $X_0^n = X_0^{n-1}$

p.188, last line $\geq \phi(\tau)$

p.190, (3.1): we must suppose that $\phi(x) \rightarrow \infty$ as $|x| \rightarrow \infty$

p.201, line -1: $M_{T+t}^{ij} - M_T^{ij} = (M_t^{ij} - M_0^{ij}) \circ \theta_T$
p.202, line 12: (two errors) accommodate Example 5.1
p.204, line 3: in the first integral: $\cdot dX_s$ instead of $\cdot X_s$
p.209, (6.2) Theorem. the displayed assumption concerns $h(x)$ not $a(x)$
p.223, lines -5 and -6: X_t , not B_t
p.230, line 7: To see that
p.258, displayed formula in (3.8): $E_x f(X_t) = \int p_t(x, y) f(y) dy$
p.279, (2.5): generated
p.287, L_{n_k}
p.288, (5.2), $|\xi_i| > M$ in (i) and (ii)
p.294, Example 6.2. For the pointwise limit to be identically 0 then the definition should be modified so that $g_n(t) = 1$ when $t \in [(n-1)/2n, 1/2)$
p.295, line 1: define $d_0(f, g)$
p.296, (ii) in (6.4): no ' on w_δ
p.305, computation of $b^{1/n}$ and $a^{1/n}$ second fraction should have a + in numerator not -.
p.317, line -3: Exercise 3.11
p.321, line 12: (missing bracket) $E(\langle X \rangle_{t \wedge T})$

References, Aizenman and Simon: No "a" in the title
References, Blumenthal and Gettoor: No "their" in the title
References, Cielsielski and Taylor: "First passage times and *sojourn* times for Brownian motion in space and *the* exact Hausdorff measure *of the sample path*"
References, Tanaka: "Note on continuous additive functionals of *the* 1-dimensional Brownian *path*"
References, J. Walsh: Astérisque
References, Yamada and Watanabe: "On the uniqueness *of...*"

Typos contributed by: Peter Calabrese, Ilya German, Pete Kiessler, Ruediger Kiesel, Kouros Owzar, Byron Schmuland, Todd Williams