

## Typos in Probability: Theory and Examples, 4th Edition

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Page numbers are those of the printed book.

### Chapter 1

Page 2, proof of (ii) in Theorem 1.1.1. Two errors:

$$B_n = A'_n - \cup_{m=1}^{n-1} A'_m \text{ (no } c)$$

Actually we are using part (ii) the definition of measure: countable additivity.

Page 3, Theorem 1.1.2. the first  $\mu((a, b])$  is missing a (.

Page 6, definition of  $F$  third line: if  $x_2 \geq 1$  and  $0 \leq x_1 < 1$ . (not  $0 \leq x_1 < 1$ ).

Page 18. Proof of Lemma 1.4.2.  $\varphi$  becomes  $\phi$  in proof. This happens a number of the times in the chapter. All the  $\phi$  have now been changed to  $\varphi$ .

### Chapter 2.

Page 49. Two lines after (2.1.1):  $e^{-\lambda}$  should be  $e^{-\lambda x}$

Page 49. Proof of 2.1.12. Middle of page 49. When we multiply by  $e^{-x}$  we integrate it as it is.

Page 49. End of proof of Theorem 2.1.12. Inside integral should be  $\int_y^\infty$ .

Page 58, near the end of the proof of Lemma 2.2.5.  $n - j + 1 \rightarrow n - k + 1$  twice.

Page 58, last line. This equation should be marked as (\*)

Page 61, middle. Improvement suggested by Carl Mueller. Let  $g_n(y) = g(ny)$ . Since  $g_n$  is bounded and  $\rightarrow 0$  a.s., we have  $(1/n) \int_0^n g(y) dy = \int_0^1 g_n(x) dx \rightarrow 0$ .

Page 61, Remark after Theorem 2.2.9. “so the assumption in **Theorem 2.2.7** is not”

Page 64, Exercise 2.2.8. (5.5) should be Theorem 2.2.6.

Page 69, line 4 of Example 2.3.2. athlete (sp)

Page 74, line 1 of the proof of Lemma 2.4.4. We being  $\rightarrow$  We begin

Page 75, line 2 of the proof of Theorem 2.4.5.  $S_i^M \rightarrow S_n^M$

### Chapter 3

Page 102, second line of proof of Theorem 3.2.4.  $f(g(Y_\infty))$  is missing one ). The same error appears two lines later.

Page 102, part (iv) of Theorem 3.2.5. For all **Borel** sets  $A$

Page 104, line -2. (sp) distribution.

Page 107, line 1. (sp) charateristic

Page 139, proof of Berry Esseen theorem. [ERROR] Two people, Christophe Leurida and Lutz Mattner, have independently pointed out that in my proof Lemma 3.4.11 is applied to

distributions  $F_L$  and  $G_L$  that do not have finite mean. I am told that the proof in Feller volume II, which I copied from, does not have this mistake.

Page 150. (3.6.1) No 2. Total variation distance is defined as 1/2 the  $L^1$  norm.

Page 158. (3.7.1)  $i$  not 1 in subscript  $P(X_i > x) = P(X_i < -x) = x^{-\alpha}/2$  for  $x \geq 1$

## Chapter 4

Page 184, proof of (4.1.1). This is not said correctly “Applying Theorem 4.1.3 to  $N = T_{n-1}$ , we see that conditional on  $T_{n-1} < \infty$ ,  $T(\theta^{T_{n-1}}) < \infty$  has the same probability as  $T < \infty$ , so

Page 188, proof of Theorem 4.1.6.  $\sum_{k=m+1}^n P(T \geq k)$  not  $P(T \geq n)$ .

Page 201, Green’s function constant is 1.516386059152... The one in the book ends with 137

## Chapter 5

Page 225. Example 5.1.5.  $\varphi$  becomes  $\phi$  in the proof.

Page 226. Theorem 5.1.2. Need to assume in (a) and (b) that  $E|X|, E|Y| < \infty$ .

Page 228. Proof of Theorem 5.1.5. Missing  $)$  in  $\int_A E(X|\mathcal{G}) dP$

Page 247. Theorem 5.3.9.  $\phi$  should be  $\varphi$

Page 266.  $\alpha$  is the number of votes for  $A$  and  $\beta$  the number of votes for  $B$ . We should assume  $\alpha > \beta$  or write  $(\alpha - \beta)^+$ .

Page 271. *Simpler proof due to Nate Eldredge.*  $X_{N \wedge n}$  is a supermartingale so by Fatou’s lemma

$$EX_0 \geq \liminf_{n \rightarrow \infty} EX_{N \wedge n} \geq EX_N$$

Page 271. Theorem 5.7.7. Another case where  $\phi = \varphi$ .

Page 273, problem 5.7.6. (LaTeX)  $P(S_T \leq a)$  not  $P(S_T \leq a)$ . A much worse problem is that to make this exercise work one needs to assume that the  $\xi_i$  are bounded below so  $Y_n = X_{n \wedge T}$  is bounded.

Page 273, Problem 5.7.7.  $E\xi_i > 0$  not  $EX_i > 0$ . However there is the much worse problem that the result you are asked to prove is incorrect. Replace the last sentence by: Let  $S_n = S_0 + \xi_1 + \dots + \xi_n$  and  $T_0 = \inf\{m : S_m = 0\}$ . Use the martingale  $X_n = \exp(\theta_0 S_n)$  to conclude that if  $S_0 = k$  then  $P(T_0 < \infty) = \exp(-\theta_0 k)$ .

## Chapter 6

Page 275. (sp) Komogorov’s extension theorem

Page 291. Seven State Example. Two errors:

$\rho_{34} > 0$  and  $\rho_{43} = 0$  so 3 is transient.

To make the graph correct we need  $p(6, 4) > 0$ .

## Chapter 7

Page 335, Exercise 7.2.3. Use **Theorem 7.2.3** and ... proof of Theorem **7.2.1** to

Page 343, Example 7.4.2. Theorem 7.4.2, not (6.1)

Page 343, Example 7.4.3.  $Y_1, Y_2, Y_3, \dots$ , let  $L_{m,n} =$  (was not a sentence)

Page 352, Exercise 7.5.4. The water starts at  $(0, 0)$ .

## Chapter 8

Page 356, right after Theorem 8.1.1. Kolmogorov's extension theorem is Theorem A.3.1, not (7.1) in the Appendix

Page 377, first line of proof of Theorem 8.5.4.  $B_t^2 = (B_s + B_t - B_s)^2$ . Second subscript was 2.

Page 378, Theorem 8.5.7.  $E_0 \exp(-\lambda T_a)$  ( $a$  should be subscript).

Page 378, Exercise 8.5.3. In part (ii),  $T$  should be  $\sigma$ .

Page 379, proof of Theorem 8.5.9: (i) There are some calculation errors in the computation of the partial derivatives of  $p_t$ . (ii) All that is shown is that  $t \rightarrow E_x u(t, B_t)$  is constant. A little more work is needed to conclude that  $E(u(t, B_t) | \mathcal{F}_s) = u(s, B_s)$ . One can do this by noting that  $v(r, x) = u(s+r, x)$  satisfies the heat equation and then use the Markov property.

Page 380, Exercise 8.5.6. The conclusion should be  $\leq 1$  not  $\leq 1/\sqrt{2}$ .

Page 388, Example 8.6.5. The trouble starts in the second formula which should be

$$|x^k - y^k| \leq \int_x^y k|z|^{k-1} dz \leq \epsilon k M^{k-1}$$

In the definition of  $G_n(M)$  we should now insist  $\max_{m \leq n} |X_m| \leq M^{-k} \sqrt{n}$  to get  $\leq k/M$  on the right-hand side of the next equation.

## Appendix.

Page 405. End of proof of Lemma A.1.6.  $\mu^*(F \cap A^c)$ .

Page 407, proof of part (ii). LaTeX error. Should be  $B = \cup_{i=1}^n$  not  $\cup i = 1^n$

Page 406. Line 2 of proof of (iii) of Theorem A.2.1.  $C \supset E$  not  $B \supset E$

Index. Law of the iterated logarithm is on page 396