### Hodge Theory, Moduli and Representation Theory

(Stony Brook, August 14-18, 2017)

<table>
<thead>
<tr>
<th>Time</th>
<th>Monday</th>
<th>Tuesday</th>
<th>Wednesday</th>
<th>Thursday</th>
<th>Friday</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-10:50</td>
<td>Pearlstein</td>
<td>Griffiths</td>
<td>Corti</td>
<td>Filip</td>
<td>Totaro</td>
</tr>
<tr>
<td>11:10-12</td>
<td>Zhang</td>
<td>Green</td>
<td>van Straten</td>
<td>Saccà</td>
<td>de Cataldo</td>
</tr>
<tr>
<td>1:30-2:20</td>
<td>Goldring</td>
<td>Wang</td>
<td>Robles</td>
<td>Schaffler</td>
<td>Schreieder</td>
</tr>
<tr>
<td>2:40-3:30</td>
<td>Flappan</td>
<td>Klingler</td>
<td>Greer</td>
<td>Hacking</td>
<td>Brosnan</td>
</tr>
<tr>
<td>4-4:50</td>
<td>Kovács</td>
<td>Kerr</td>
<td>Maulik</td>
<td>Kollár</td>
<td><strong>departure</strong> Fri@3:30</td>
</tr>
</tbody>
</table>

**Note:** Talks will take place at Simons Center, Room 103.
Patrick Brosnan – *Palindromic Betti numbers.*
A topological space of dimension $d$ has palindromic Betti numbers if the Betti numbers are symmetric about the middle dimension. In other words, numerically, the space satisfies Poincare duality. I will explain a few consequences of palindromic cohomology in special fibers of families of varieties. The first is a joint theorem with Tim Chow relating palindromic cohomology of a fiber to local invariant cycles. Essentially, it says that the local invariant cycle map of Beilinson, Bernstein and Deligne is an isomorphism if and only if the special fiber has palindromic cohomology. The second is a criterion for certain families of abelian varieties (arising in the work of Laza, Sacca and Voisin) to have irreducible fibers.

Alessio Corti – *Laurent polynomial of small ramification.*
I will define the ramification of a local system on a topological surface (after Golyshev). Then I will show how to compute the ramification of $\mathbb{R}^n \mathbb{f};\mathbb{Z}$ of a Laurent polynomial $f$ in $n + 1$ variables and explain how to construct many examples of Laurent polynomials with exceptionally low ramification.

Mark de Cataldo – *Supports for Hitchin fibrations.*
Hitchin fibrations are remarkable families of (degenerate) abelian varieties which appear in geometry, arithmetic, representation theory, etc. The study of the singularities of these maps has been the focus of much work, at least since Ngo’s proof of the fundamental lemma. One way to encode some of these singularities is to look at the summands appearing in the decomposition theorem. In this talk, we focus on the subvarieties of the target of the Hitchin map that support such summands. I will report on work in progress with J. Heinloth and L. Migliorini in the case of $\text{GL}(n)$.

Simion Filip – *Hodge theory from far away.*
I will discuss some coarse properties of Hodge-theoretic invariants. For example, what can one say about the large-scale behavior of the period map? If the monodromy is arithmetic, or of infinite index in an arithmetic group, how does this affect the period map? Some of the tools to address these questions come from dynamical systems and I will give an introduction to the relevant concepts.

Laure Flapan – *Monodromy groups of Kodaira fibrations.*
A Kodaira fibration is a non-isotrivial fibration $f: S \to B$ from a smooth algebraic surface $S$ to a smooth algebraic curve $B$ such that all the fibers are smooth algebraic curves. Such fibrations thus arise as complete curves inside the moduli space $\mathcal{M}_g$ of genus $g$ algebraic curves. We investigate the possible connected monodromy groups of a Kodaira fibration in the case $g = 3$ and classify which such groups can arise from a Kodaira fibration obtained as a general complete intersection curve inside a subvariety of $\mathcal{M}_3$.


I will talk about our program that aims to connect (A) Automorphic Algebraicity, (B) G-Zip Geometricity, and (C) Griffiths-Schmid Algebraicity. It was initiated jointly with J.-S. Koskivirta and developed further together with B. Stroh and Y. Brunebarbe. In a nutshell, the theme of the program may be summed up as "characteristic-shifting between automorphic spaces". The focus of the talk will be to illustrate the analogy between stacks of G-Zips in characteristic $p$ and Mumford-Tate domains over $\mathbb{C}$. I will explain how the pursuit of the analogy leads to new results and conjectures, both in characteristic $p$ and over $\mathbb{C}$. Two examples include: (1) Proof and generalization of a conjecture of F. Diamond on mod $p$ Hilbert modular forms, (2) Extension of some results of Griffiths-Schmid on positivity of automorphic bundles.

Mark Green – *Three Constructions Associated to a Nilpotent Orbit.*

This talk, a follow-up to the one by Phillip Griffiths, will survey some recent joint work with P. Griffiths, R. Laza and C. Robles. One of the overall themes of our work has been multi-parameter families of varieties and how the variation of their Hodge structures reflects their geometry, in particular, the relationship between the singularities the varieties acquire and the degeneration of their Hodge structures. Associated to such a multiparameter family is a nilpotent orbit over a product of punctured discs. Some interesting algebraic invariants of such nilpotent orbits we have been studying are a semisimple Lie algebra, a monomial map that fills in across the punctures, and a metric on the determinants of the Hodge filtration bundles $det(F^p)$.

Francois Greer – *Modular Curve Counts from Noether-Lefschetz Theory.*

Let $X$ be a smooth projective variety fibered by genus one curves. Physical heuristics imply that the Gromov-Witten generating functions for $X$ should be modular forms. We prove this in several cases, using the Hodge theory of elliptic surfaces and the cohomological theta correspondence to extract enumerative counts of smooth rational curves over lines in the base.

Phillip Griffiths – *Hodge Theory and Moduli.*
In this talk and the subsequent one by Mark Green we will describe and give examples of the Hodge-theoretic Satake-Baily-Borel completion $\overline{M}$ of the image $M$ of a period mapping

$$\Phi : B \rightarrow \Gamma \backslash D$$

where $B$ is a smooth quasi-projective variety. We are primarily concerned with the non-classical situation where $D$ is not a Hermitian symmetric domain, in which case new and interesting phenomena arise. The main result is the construction of $\overline{M}$ as a compact analytic variety and that the extended Hodge line bundle $\Lambda_e \rightarrow \overline{M}$ is ample. Applications include the use of the extended period mapping for the study of $\Phi_e : \overline{M} \rightarrow \overline{M}$ where $\overline{M}$ is the Alexeev-Kollár-Shephard-Baron moduli space for a given class of surfaces of general type with $p_g = 2$. Using Lie theory, and the above result and some analytic geometry, the boundary $\partial M = \overline{M} \setminus M$ is well understood. One seeks to use this together with $\Phi_e|_{\partial M}$ as a guide to the understanding of $\partial M$. This objective will be illustrated for a class of algebraic surfaces.

**Paul Hacking** – *Smoothing toric Fano varieties via mirror symmetry.*

A pair $(X, E)$ consisting of a smooth Fano variety together with a choice of smooth anticanonical divisor corresponds under mirror symmetry to a quasiprojective variety $U$ together with a regular function $W$ on $U$ such that the fibers of $W$ are proper Calabi-Yau varieties and the monodromy at infinity is maximally unipotent. Moreover, a degeneration of $(X, E)$ to a singular toric Fano variety together with its toric boundary corresponds to an open embedding of an algebraic torus in $U$. Using this heuristic, we construct smoothings of Gorenstein toric Fano 3-folds determined by combinatorial data encoding the construction of the mirror as a blowup of a toric variety.

This is a part of a program initiated by Coates, Corti, Galkin, Golyshev, and Kasprzyk to classify smooth Fano varieties using mirror symmetry. Period and Gromov–Witten calculations by these authors suggest that all deformation types of Fano 3-folds are obtained by our construction.

This is a report on work in progress with Alessio Corti, Mark Gross, and Andrea Petracci.

**Matt Kerr** – *Admissible nilpotent cones.*

Nilpotent cones in reductive Lie algebras are the basic linear-algebraic gadget underlying Hodge-theoretic compactifications of moduli spaces. They reflect the possible monodromies of multiparameter degenerations of varieties (over a product of punctured disks) and their attached variations of Hodge structure. In this talk, we describe results with C. Robles and G. Pearlstein which provide a combinatorial algorithm for classifying nilpotent cones up to a suitable equivalency, relate this to geometry in a couple of examples, and discuss the relation to Looijenga-Lunts groups.
Bruno Klingler – *Hodge theory and atypical intersections.*

Given a variation of Hodge structures $V$ over a smooth quasi-projective base $S$, I will explain the notion of an atypical subvariety for $(S, V)$ and state a simple general conjecture about these atypical subvarieties. When $S$ is a Shimura variety and $V$ a standard variation of Hodge structure on $S$, one recovers the Zilber-Pink conjectures, in particular the André-Oort conjecture.

Sándor Kovács – *Local base change for stable families*

We prove that for a flat finite type family whose fibers have Du Bois singularities, the cohomology sheaves of the relative dualizing complex are flat and commute with base change.

János Kollár – *Moduli of varieties of general type.*

Davesh Maulik – *Gopakumar-Vafa invariants via vanishing cycles.*

Given a Calabi-Yau threefold $X$, one can count curves on $X$ using various approaches, for example using stable maps or ideal sheaves; for any curve class on $X$, this produces an infinite sequence of invariants, indexed by extra discrete data (e.g. by the domain genus of a stable map). Conjecturally, however, this sequence is determined by only a finite number of integer invariants, known as Gopakumar-Vafa invariants. In this talk, I will propose a direct definition of these invariants via sheaves of vanishing cycles, building on earlier approaches of Kiem-Li and Hosono-Saito-Takahashi. Conjecturally, these should agree with the invariants as defined by stable maps. I will also explain how to prove the conjectural correspondence in various cases. This is joint work in progress with Yukinobu Toda.

Gregory Pearlstein – *Mixed Hodge Metrics.*

I will give an overview of recent work with P. Brosnan on the asymptotic behavior of archimedean heights, and with C. Peters on the differential geometry of mixed period domains.

Colleen Robles – *Generalizing the Satake-Baily-Borel compactification to arithmetic quotients of Mumford-Tate domains.*

The SBB compactification is an projective algebraic completion of a locally Hermitian symmetric space. This construction, along with Borel’s Extension Theorem, provides the conduit to apply Hodge theory to study moduli, and their compactifications, of principally polarized abelian varieties and K3 surfaces (and, more generally, any moduli space of smooth projective varieties for which the corresponding Mumford-Tate domain is Hermitian).
Most period (and Mumford-Tate) domains are not Hermitian, and so one would like to generalize SBB (and the extension theorem) in the hopes of similarly applying Hodge theory to study the corresponding moduli and their compactifications.

Despite the robust Hodge theoretic interpretation and applications, SBB is a group theoretic construction. This suggests two natural generalizations: GT-SBB and HT-SBB; the first is group theoretic in nature, the second is Hodge theoretic. While the two perspectives coincide when D is Hermitian, they differ when D is non-Hermitian (for both group theoretic and Hodge theoretic reasons). HT-SBB compactifies the image of a period map and admits an extension theorem. GT-SBB is a "horizontal completion" of the ambient space, and is a "meta-construction" encoding the structures that are universal among all instances of HT-SBB (for a given period/Mumford-Tate domain).

In this talk I will present the GT-SBB. (The talks of Green and Griffiths will introduce the HT-SBB and discuss applications to the study of moduli.) This is joint work with Mark Green, Phillip Griffiths and Radu Laza.

Giulia Saccà – Degenerations of hyperkähler manifolds

The problem of understanding semistable degenerations of $K3$ surfaces has been greatly studied and is completely understood (Kulikov-Pinkham-Persson). The aim of this talk is to present joint work with J. Kollár, R. Laza, and C. Voisin generalizing some of these results to higher dimensional hyperkähler (HK) manifolds. I will also present some applications, including a generalization of theorem of Huybrechts to possibly singular symplectic varieties and shortcuts to showing that certain HK manifolds are of a given deformation type.

Luca Schaffler – Toward a compactification of the moduli space of Enriques surfaces by KSBA stable pairs.

This talk is about an ongoing project that aims at constructing a compactification by KSBA stable pairs of the moduli space of Enriques surfaces. More precisely, we consider the 10-dimensional family of Enriques surfaces originally constructed by Enriques as normalizations of specific sextic hypersurfaces in $\mathbb{P}^3$. After explaining the general ideas and techniques, we apply them to the 4-dimensional subfamily of Enriques surfaces coming from "diagonal" Enriques sextics. In this case, we have a complete description of the KSBA compactification, and we relate it to the corresponding Baily-Borel compactification. We observe that the boundary of this stable pairs compactification is combinatorially equivalent to an appropriate Looijenga’s semitoric compactification. Time permitting, we discuss some recent progress toward compactifying bigger subfamilies of Enriques surfaces.
Stefan Schreieder – The rationality problem for quadric bundles.
We study the (stable) rationality problem for quadric bundles $X$ over rational bases $S$. By a theorem of Lang, such bundles are rational if $r > 2n - 2$, where $r$ denotes the fibre dimension and $n = \dim(S)$ denotes the dimension of the base. We show that this result is sharp. In fact, for any $r$ at most $2n - 2$, there are many smooth $r$-fold quadric bundles over rational $n$-folds which are not even stably rational.

Burt Totaro – Equivariant Hodge theory for varieties in positive characteristic.
There is a good notion of equivariant Hodge theory for complex varieties with an action of an algebraic group. In this talk, we define equivariant Hodge theory for varieties in positive characteristic $p$. The calculations so far have strong similarities with mod $p$ equivariant cohomology in topology.

Duco van Straten – CY-operators and L-functions.
In the talk I will report on work in progress with P. Candelas and X. de la Ossa. For a given variety and prime number, the Frobenius-operator determines the Euler-factor of the $L$-function at that prime. A fundamental insight (that goes back at least to the work of Dwork) is that if the variety varies in a family, then the variation of the Frobenius-operator is entirely controlled by the Gauss-Manin connection of the family. Using this idea and starting from a point of maximal unipotent monodromy, it appears that one can compute the Euler-factors of the fibres of a family directly from the associated Picard-Fuchs equation. I will report on some striking observations, applications and conjectures.

Botong Wang – Absolute constructible sets.
Motivated by Deligne’s definition of absolute Hodge cycles, Simpson introduced the concept of absolute constructible sets in the moduli space of local systems on a complex smooth projective variety. In this talk, I will discuss a generalization of Simpson’s absolute constructible sets. The new absolute constructible sets can be defined over any complex smooth variety and can be lifted to the derived category of constructible complexes. For rank one local systems, we have a structure theorem of absolute constructible sets, and in higher rank, we have some conjectures of Andre-Oort type. I will talk about two applications. The first is a generalization of the structure theorems of cohomology jump loci. And the second is the proof of the decomposition theorem of any rank one local system by reducing to the ones of geometric origin. This is joint work with Nero Budur.
Zheng Zhang – On the period maps for certain Horikawa surfaces and for cubic threefolds and their hyperplane sections.

It is an interesting problem to attach moduli meanings to locally symmetric domains via period maps. Besides the classical cases like polarized abelian varieties and lattice polarized K3 surfaces, such examples include quartic curves (by Kondo), cubic surfaces and cubic threefolds (by Allcock, Carlson and Toledo), and some Calabi-Yau varieties (by Borcea, Voisin, and van Geemen). In the talk we will discuss two examples along these lines: (1) certain surfaces of general type with $p_g = 2$ and $K^2 = 1$; (2) pairs consisting of a cubic threefold and a hyperplane section. This is joint work with R. Laza and G. Pearlstein.