Trigonometric Identities

1. Use the formula for $\sin(A + B)$ to find a formula for $\sin(2A)$. (Note $2A = A + A$)

2. Graph $y = \sin 2x$ and $y = 2 \sin x \cos x$ on the same set of axes.

3. Use expansion formulas to show that $\cos \left( \frac{\pi}{2} - x \right) = \sin x$. Check by graphing the two functions to see if their graphs are the same.

4. Use the formula for $\cos(A + B)$ to show that $\cos(2x) = \cos^2(x) - \sin^2(x)$. Then show that $\cos^2 x - \sin^2 x = 2 \cos^2 x - 1 = 1 - 2 \sin^2 x$.

5. Use expansion formulas for $\sin(A + B)$ and $\cos(A + B)$ to complete the following derivation of the identify $\tan(A + B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$:

6. Compute the following limits:

   (a) $\lim_{x \to 0} \frac{\sin(2x)}{x}$ (Hint: use your answer to Question 1)

   (b) $\lim_{x \to \infty} \frac{\sin(2x)}{x}$

   (c) $\lim_{x \to 0} \frac{\sin^2 x}{x}$

   (d) $\lim_{x \to 0} \frac{x}{\cos x}$

   (e) $\lim_{x \to 0} \frac{x}{\sin x}$

   (f) $\lim_{x \to 0} x \cot x$

7. Prove the following identities:

   (a) $\csc^2 x - \cot^2 x = 1$

   (b) $\frac{\sin^2 x}{\cos x} + \cos x = \sec x$

   (c) $\cos \theta \tan \theta \csc \theta = 1$