Review of Basic Trigonometry

1. For each value of $\theta$ below, draw the unit circle and the appropriate “reference triangle.” Then use this to determine the exact values of $\sin \theta$ and $\cos \theta$. (Make sure to label and/or show all of your work; do NOT use your calculator.)

(a) $\theta = \frac{5\pi}{6}$
(b) $\theta = -\frac{\pi}{4}$
(c) $\theta = 480^\circ$
(d) $\theta = \frac{\pi}{3} + 1,000,000\pi$

2. Suppose that the terminal side of an angle $\theta$ lies in Quadrant III and lies on the line $y = 3x$. Find $\sin \theta$.

3. Given the right triangle below, find the exact values of $\sin x$, $\cos x$, and $\tan x$.

4. Given the right triangle below, solve for $x$.
5. Given that \( \tan \theta = -\frac{5}{4} \) and \( \cos \theta > 0 \), find the exact values of \( \sin \theta \) and \( \cos \theta \). Show your work clearly; your work should use a sketch of the unit circle.

6. Let \( t \) be an angle in the first quadrant, as pictured below. Evaluate the following expressions in terms of \( a \).

   (a) \( \sin(t + 2\pi) = \) 
   
   (b) \( \sin(t + \pi) = \) 
   
   (c) \( \cos\left(\frac{\pi}{2} - t\right) = \) 
   
   (d) \( \sin(\pi - t) = \) 
   
   (e) \( \sin(2\pi - t) = \) 
   
   (f) \( \cos\left(\frac{3\pi}{2} - t\right) = \) 

7. True or False: In the diagram, the coordinates of point \( P \) are \((-\cos t, \sin t)\).
8. Suppose the circle pictured below has radius 8 cm. Complete the following statements.

(a) If the measure of angle \( ACB \) is 128 degrees, then the measure of angle \( ACB \) is \( \frac{2\pi}{3} \) radians and the measure of arc \( AXB \) is \( 16\pi \) cm.

(b) If the measure of angle \( ACB \) is 2 radians, then the measure of angle \( ACB \) is \( 114^\circ \) degrees and the measure of arc \( AXB \) is \( 16\pi \) cm.

(c) If the measure of arc \( AXB \) is 15 cm, then the measure of angle \( ACB \) is \( 150^\circ \) degrees and the measure of angle \( ACB \) is \( \frac{5\pi}{3} \) radians.

![Circle with points A, B, C, and X](image)

9. Find the \( x \) and \( y \) coordinates on the unit circle determined by the following angles. If possible, find these coordinates exactly. Otherwise, approximate them.

(a) 30°
(b) 315°
(c) 130°
(d) −240°
(e) 1000°

10. (a) For what values of \( \theta \) is \( \sin(\theta) \geq 0? \)

(b) For what values of \( \theta \) is \( \cos(\theta) \leq 0? \)

(c) For what values of \( \theta \) is \( \sin(\theta) \geq \cos(\theta)? \)

11. Find the exact values of the following.

(a) \( \sin \left( \frac{\pi}{3} \right) \)
(b) \( \cos \left( -\frac{3\pi}{4} \right) \)
(c) \( \sin^2(2.3) + \cos^2(2.3) \)
(d) \( \tan \left( \frac{7\pi}{6} \right) \)
Solving Triangles - Trigonometry Word Problems

1. A rocket is fired at sea level and climbs at a constant angle of 75° through a distance of 10,000 feet. Approximate its altitude to the nearest foot.

2. An airline pilot wishes to make his approach to an airstrip at an angle of 10° with the horizontal. If he is flying at an altitude of 5000 feet, approximately how far from the airstrip should he begin his descent?

3. A 16 foot long ladder is leaning against a wall and making a 60° angle with the ground. Without using your calculator determine exactly how high on the wall the top of the ladder is resting.

4. An astronomer is studying two distant stars each approximately 12 thousand light years from earth. She finds that the angle spanned by the two stars, with the earth at its vertex, is approximately 74°. Estimate the distance between the two stars.

5. A CB antenna is located on the top of a garage that is 16 feet tall. From a point on level ground that is 100 feet from a point directly below the antenna, the antenna subtends an angle of 12°. Approximate the length of the antenna.

6. From a point A that is 8 meters above level ground, the angle of elevation of the top of a building is 31° and the angle of depression of the base of the building is 12°. Approximate the height of the building.

7. A motorist, traveling along a level highway at a speed of 60 km/h directly toward a distant mountain, observes that between 1:00 p.m. and 1:10 p.m. the angle of elevation of the top of the mountain changes from 10° to 70°. Approximate the height of the mountain.

8. The center of the great clock in the bell tower of the British parliament building is approximately 250 feet above the ground. The minute hand on the clock is 14 feet long. How high above the ground is the tip of the minute hand when the clock reads ten minutes after the hour?

9. The Hubble space telescope orbits the earth every 90 minutes in a near circular orbit 370 miles above the surface of the earth. (The radius of the earth is 3960 miles.)
   (a) What distance along its orbit will the Hubble telescope travel in 1 hour?
   (b) What is the radian measure of the angle through which the Hubble telescope will move after traveling 1000 miles along its orbit? (The angle is measured from the center of the earth.)
Graphing and Transforming Trigonometric Functions

1. Use Geogebra to help you sketch the following graphs on the axes provided. The domain of the graphs should be $[-2\pi, 2\pi]$. Be sure to label your axes.

   (a) $y = \sin(x)$, $y = \frac{1}{2}\sin(x)$, $y = -3\sin(x)$

   (b) $y = \cos(x)$, $y = \cos(2x)$, $y = \cos\left(\frac{1}{3}x\right)$

   (c) $y = \sin(x)$, $y = \sin\left(x - \frac{\pi}{4}\right)$, $y = \sin\left(x + \frac{\pi}{2}\right)$
2. Match the following transformations with the description of their effects. (Assume \( k > 1 \)).

   (a) \( f(x - k) \)  ______ Vertical shift \( k \) units down
   (b) \( f(x) + k \)  ______ Horizontal stretch by a factor of \( k \)
   (c) \( f \left( \frac{1}{k}x \right) \)  ______ Vertical shrink by a factor of \( k \)
   (d) \( kf(x) \)  ______ Horizontal shift \( k \) units to the left
   (e) \( f(x + k) \)  ______ Vertical shift \( k \) units up
   (f) \( f(x) - k \)  ______ Horizontal shrink by a factor of \( k \)
   (g) \( f(kx) \)  ______ Vertical stretch by a factor of \( k \)
   (h) \( \frac{1}{k}f(x) \)  ______ Horizontal shift \( k \) units to the right

3. Sketch the following graphs by hand. The range of the graphs should be \([-2\pi, 2\pi]\). Be sure to label your axes.

   (a) \( y = 3 \sin(2x) \)

   ______________________

   (b) \( y = -\frac{1}{2} \cos \left( x - \frac{\pi}{2} \right) \)

   ______________________
4. Find possible formulas for the trigonometric functions graphed below.

(a) \( y = \cos\left(\frac{1}{3}x\right) - 1 \)

(b) \( y = |3\sin(2x)| \)