Composition of Functions

1. (a) Suppose \( f(x) = x^2 + 1 \) and \( g(x) = 2x - 1 \). Write simplified expressions for \( f(g(x)) \) and \( g(f(x)) \).

(b) Suppose \( f(x) = \frac{1}{x} \) and \( g(x) = \frac{1}{x^2 + 1} \). Write simplified expressions for \( f(g(x)) \) and \( g(f(x)) \).

(c) Suppose \( f(x) = \frac{x}{x^2} \) and \( g(x) = \sqrt{x^2 + x} \). Write simplified expressions for \( f(g(x)) \) and \( g(f(x)) \).

(d) Suppose \( f(x) = \frac{x + 1}{x - 1} \). Write a simplified expression for \( f(f(x)) \).

(e) Suppose \( f(x) = \sqrt{x + 1} \) and \( g(f(x)) = \frac{1}{3x + 1} \). Find an expression for \( g(x) \).

(f) Suppose \( f(x) = x + 1 \) and \( g(f(x)) = \frac{1}{x^2 - 1} \). Find an expression for \( g(x) \).

2. Let \( f(x) = x^2 - 4 \) and let \( g(x) = \sqrt{x} \). Find the domain of the function \( h(x) = g(f(x)) \).

3. The table below contains certain values of the functions \( h \) and \( k \). Fill in the table with values for the function \( g(x) = h(k(x)) \).

<table>
<thead>
<tr>
<th>( x )</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( k(x) )</td>
<td>1</td>
<td>0</td>
<td>3</td>
<td>-2</td>
<td>2</td>
<td>-1</td>
</tr>
<tr>
<td>( h(x) )</td>
<td>-3</td>
<td>-2</td>
<td>-1</td>
<td>5</td>
<td>7</td>
<td>2</td>
</tr>
<tr>
<td>( g(x) )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

4. (a) Let \( T(t) \) be the average daily temperature (°F) in Durham, North Carolina, at time \( t \) (days from January 1). Let \( E(x) \) be the amount of electricity used (kilowatts) by the residents of Durham when the average daily temperature is \( x \) degrees. Describe the meaning of the function \( G(t) = E(T(t)) \) in practical terms. Include units in your answer.

(b) Let \( v(t) \) be the speed (meters per second) of a marathon runner \( t \) minutes from the start of the race. Let \( O(x) \) be the amount of oxygen (liters/min) required by this runner to maintain a speed of \( x \) meters per second. Explain the meaning of the function \( f(t) = O(v(t)) \) in practical terms. Include units in your answer.

(c) Let \( L(e) \) be the life expectancy (years) of a person with \( e \) years of education. Let \( E(i) \) be the average number of years of education obtained by people whose parents have an average annual income of \( i \) dollars. Describe the function \( f(i) = L(E(i)) \) in practical terms. Include units in your answer.
5. The graphs of the functions $f$ and $g$ are given below. Sketch the graphs of $f(g(x))$ and $g(f(x))$. 

![Graphs of functions f and g]