105L Worksheets: Compound Interest

Compound Interest

**Purpose**  The purpose of this worksheet is to develop formulas for computing interest and to examine the effect on the return on an investment when interest rates and the number of compounding periods each year are varied.

**Part I: Introductory Exercises**

Make sure you carefully show and explain how you arrived at your answers. For these problems you are *developing* a formula so you shouldn’t be using a formula here only common sense.

1. Suppose you invest $10 at 6% annual interest, compounded annually.
   
   (a) How much money would you have after one year?
   (b) How much money would you have after two years?
   (c) How much money would you have after ten years?
   (d) How much money would you have after $t$ years?

2. Suppose instead you invest $10 at 6% annual interest, compounded semi-annually (i.e., twice a year). That means that each half-year you add 3% to what you have accumulated.
   
   (a) How much money would you have after one year?
   (b) How much money would you have after two years?
   (c) How much money would you have after ten years?
   (d) How much money would you have after $t$ years?

3. If we double the initial investment and make an investment of $20, how much will we have after 10 years if the return is 6%, compounded annually? after 100 years?

4. Now suppose we double the interest rate to 12%, compounded annually, but make only a $10 initial investment? How much will we have after 10 years? after 100 years?

**Part II: Pushing the Limit**

5. Suppose we invest $1 at 100% interest. How much will we have after one year if the interest is compounded
   
   (a) monthly?
   (b) daily?
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(c) every minute?

For monthly compounding, $n$, the number of compounding periods per year is 12. For
daily compounding $n = 365$ and for compounding every minute $n = 525600$. If we let
$n \to \infty$, we say the investment is compounded continuously.

(d) How much will we have after one year if the interest is compounded continuously? Ex-
periment with your calculator using larger values of $n$.

(e) As $n$ increases, does the value of the investment increase? Does it increase without
bound, that is, will the value of the investment eventually reach any amount as $n$ in-
creases?

6. Suppose we invest $1 at an interest rate of 0.05 (i.e., 5%). How much will we have after one
year if the interest is compounded continuously? Compare your result to $e^{0.05}$.

7. Suppose we invest $1 at an interest rate of $r$. How much will we have after 1 year if the
interest is compounded continuously? How much will we have after $t$ years if the interest is
compounded continuously?

Part III: Exercises

8. A bank advertises the following rates for its certificates of deposit (a deposit which, if interest
is to be earned, must be left in the bank until the term of the deposit is completed):

<table>
<thead>
<tr>
<th>time</th>
<th>annual percentage (compounded daily)</th>
<th>effective yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>5.00%</td>
<td>5.13%</td>
</tr>
<tr>
<td>$1 \frac{1}{2}$ years</td>
<td>5.05%</td>
<td>5.25%</td>
</tr>
<tr>
<td>5 years</td>
<td>6.34%</td>
<td>7.46%</td>
</tr>
</tbody>
</table>

Define effective yield in a manner consistent with the table above. Explain clearly (using
complete sentences) what you think is meant by effective yield and how your definition is
consistent with the given data.

9. Which is a better investment: an investment at a 5.25 % annual rate compounded once a
year or an investment at a 5 % continuous rate? Explain.

10. Suppose you invest, for one year, $100 in a account with an interest rate of $r$ compounded
continuously. At the end of the year you would have $100e^r$ dollars. If the interest rate is
doubled to $2r$ (and still compounded continuously), how much would you have to invest to
get the same yield of $100e^r$ dollars?

11. You are about to invest in an account that pays 5% compounded continuously. How much
would you have to invest now so that in 10 years you would have $20,000.
1. (a) $10.06
   (b) $11.24
   (c) $17.91
   (d) $10 \cdot 1.06^t
2. (a) $10.61
   (b) $11.26
   (c) $18.06
   (d) $10 \cdot 1.03^{2t}
3. $35.82; $6,786.04.
4. $31.06; $835,222.66.
5. (a) $2.61
   (b) $2.71
   (c) $2.72
   (d) $2.72
   (e) Yes; No.
6. $1.05
7. \[ \lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^n = e^r \]; \[ \lim_{n \to \infty} \left(1 + \frac{r}{n}\right)^{nt} = e^{rt} \]
8. Discuss with your teacher.
9. 5.25% is better.
10. $100e^{-r}$.
11. $12,130.61$. 