## A Crash Course in Trigonometry Part I

## Right-Angled Triangles

Consider two right-angled triangles with one identical angle (other than the right angle):


1. (a) What can you say about the third angle in each of the triangles?
(b) Therefore, the two triangles are $\qquad$ .
(c) This implies that:

$$
\frac{o}{h}=\ldots, \text { and } \frac{a}{h}=\ldots, \quad \frac{o}{a}=
$$

Conclusion: The above ratios only depend on $\qquad$ .

Definitions - Basic Trig Functions: given an angle $\theta$ in a right-angled triangle, we define the following three functions:

$$
\sin \theta=\frac{\text { opp }}{\text { hyp }}, \cos \theta=\frac{\text { adj }}{\text { hyp }}, \tan \theta=\frac{\text { opp }}{\text { adj }},
$$

where opp, and adj are the lengths of the sides of the corresponding right-angled triangle positions opposite, and adjacent to angle $\theta$ respectively, and hyp is the length of the hypotenuse.
2. For what angles are these functions currently defined? i.e. What are their domains?

## Special Values of the Trig Functions

3. By finding the value of $x$ in the following $45^{\circ}-45^{\circ}-90^{\circ}$ triangle exactly (no decimals!), compute the values below:


$$
\begin{aligned}
& \sin 45^{\circ}= \\
& \cos 45^{\circ}= \\
& \tan 45^{\circ}=
\end{aligned}
$$

4. By finding the value of $h$ (i.e. the length of the dashed line) in the following $60^{\circ}-60^{\circ}-60^{\circ}$ triangle exactly (no decimals!), compute the values below:


$$
\begin{aligned}
& \sin 60^{\circ}= \\
& \cos 60^{\circ}= \\
& \tan 60^{\circ}=
\end{aligned}
$$

You can also use the same triangle to compute the following values (rotate the page $90^{\circ}$ ):

$$
\sin 30^{\circ}=\zeta_{\square}, \cos 30^{\circ}=, \tan 30^{\circ}=\ldots .
$$

## Extending the Domains

5. (a) Suppose that the hypotenuse of a right-angled triangle has length 1. Draw such a triangle on the axes to the right, with its angle $\theta$ located at the origin, and the adjacent edge on the $x$-axis. Imagine the angle increasing from $0^{\circ}$ to $90^{\circ}$. Why does it trace out a quarter cirlce?
(b) Now fix the angle $\theta$, and label the corresponding point on your traced shape $(x, y)$. Then

$$
\cos \theta=
$$

$\qquad$ and $\sin \theta=$ $\qquad$

6. (a) Continue drawing your shape all the way around on the next set of axes. Label an angle with $90^{\circ}<\theta<180^{\circ}$.

For such an angle, we define $\cos \theta=x$, and $\sin \theta=y$, where $x$ and $y$ are the coordinates of the point on the unit circle corresponding to the angle $\theta$, as measured anti-clockwise from the positive horizontal axis.
(b) For angles $90^{\circ}<\theta<180^{\circ}$, is $\sin \theta$ positive or negative? What about $\cos \theta$ ?

## Definitions - Trig Functions for General Angles

Given any angle $\theta, \sin \theta$ is the $y$-coordinate of the point on the unit circle whose corresponding radius makes the angle $\theta$ with the positive horizontal axis, measure anti-clockwise. $\cos \theta$ is the $x$-coordinate of the same point. To get negative angles, measure clockwise instead.

For an angle $\theta$, and the corresponding point on the unit circle $(x, y)$,

$$
\tan \theta=\underline{ }=
$$

## Homework: Solving Triangles and Word Problems

1. Given the right triangle to the right, find the exact values of $\sin x, \cos x$, and $\tan x$.

2. Given the right triangle to the right, solve for $x$.


For all the problems below, draw a good picture first, then solve. Write your answers on a separate sheet of paper.
3. A rocket is fired at sea level and climbs at a constant angle of $75^{\circ}$ through a distance of 10,000 feet. Approximate its altitude to the nearest foot.
4. An airline pilot wishes to make his approach to an airstrip at an angle of $10^{\circ}$ with the horizontal. If they are flying at an altitude of 5000 feet, approximately how far from the airstrip should they begin their descent?
5. A 16 foot long ladder is leaning against a wall and making a $60^{\circ}$ angle with the ground. Without using your calculator determine exactly how high on the wall the top of the ladder is resting.
6. An astronomer is studying two distant stars each approximately 12 thousand light years from earth. They find that the angle spanned by the two stars, with the earth at its vertex, is approximately $74^{\circ}$. Estimate the distance between the two stars.
7. From a point $A$ that is 8 meters above level ground, some distance away from a building, the angle of elevation of the top of the building is $31^{\circ}$ and the angle of depression of the base of the building is $12^{\circ}$. Approximate the height of the building. (Hint: you will need two equations, as you have two unknowns!)

