

Inverse Functions

Purpose: In this lab we will learn and practice the concept of *inverse functions*.

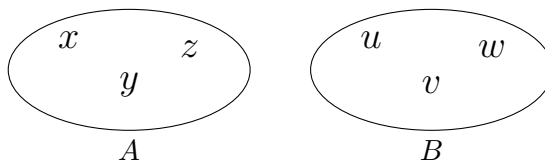
Part I: Invertible Functions

Suppose f is a function and that there is a corresponding function g such that $g(f(t)) = t$ and $f(g(t)) = t$ for every value of t in the domains of f and g . Then g is called the *inverse* function of f and is denoted f^{-1} . (This notation can be confusing. Note that $f^{-1} \neq \frac{1}{f}$!) Another way of saying this is that:

$$f(f^{-1}(x)) = x = f^{-1}(f(x)).$$

1. Suppose g is a function with domain $A = \{x, y, z\}$ and range $B = \{u, v, w\}$ such that $g(x) = u$, $g(y) = v$ and $g(z) = w$.

- (a) Draw arrows from A to B that depict what function g does.

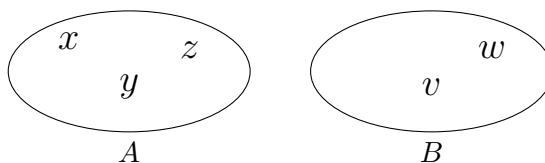


- (b) Describe $g^{-1}(t)$. Remember that $g^{-1}(g(t)) = t$ for every element in the domain of g . So we want, in the picture above $g^{-1}(g(x)) = x$, $g^{-1}(g(y)) = y$, and $g^{-1}(g(z)) = z$.
- (c) What is the domain of g^{-1} ? What is its range?
- (d) Is g^{-1} a function?

$$\text{Domain of } g = \text{Range of } g^{-1} \text{ and Domain of } g^{-1} = \text{Range of } g.$$

2. Suppose h is a function with domain $A = \{x, y, z\}$ and range $B = \{u, v\}$ such that $h(x) = v$, $h(y) = v$ and $h(z) = w$. Is h a function?

- (a) Draw arrows from A to B that depict what function h does.



- (b) Can you describe an inverse for h ? Why or why not?

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Note that in the function g from Question 1, each element in the domain is assigned exactly one element in the range and each element in the range is assigned exactly one element in the domain. This is called a *one-to-one* function.

3. Is h (from Question 2) a one-to-one function?

If f^{-1} is a function, we say that f is *invertible*.

4. Is g an invertible function? What about h ? Carefully explain why each of the following conditions ensures that a function f is invertible:

Invertible Functions

- A function f is invertible if and only if it is a one-to-one function.
- That is, if $x \neq y$ then $f(x) \neq f(y)$.
- In other words, a function is invertible if and only if its graph passes the *horizontal line test*.

Part II: Computing Inverses and Drawing their Graphs

5. (a) If $y = f(x)$, what is $f^{-1}(y)$?
(b) If $x = f^{-1}(y)$, what is $f(x)$?

Computing Inverses

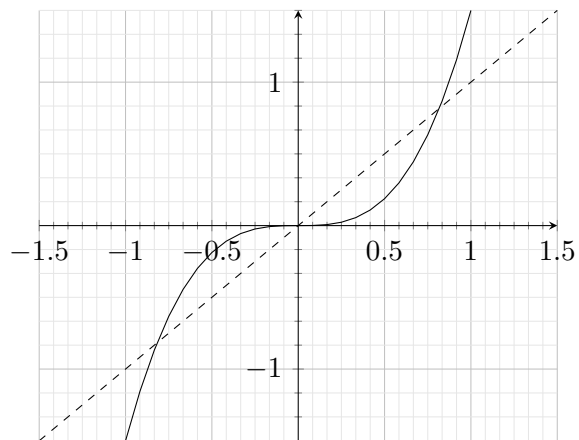
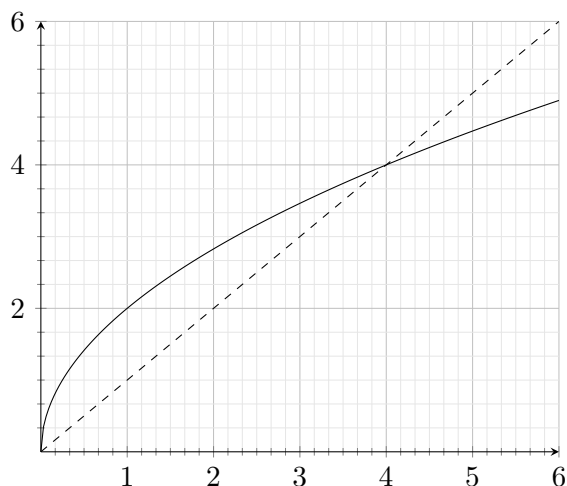
Domain of f = Range of f^{-1} and Domain of f^{-1} = Range of f .

So given f we can sometimes find f^{-1} by switching the domain and range (that is, the x and y), and solving for y .

6. Graph the function $f(x) = 2x + 3$. Is it invertible? If so, can you find a formula for its inverse, f^{-1} ? If so, graph it on the same axes.
7. Graph the function $f(x) = \frac{x}{x+1}$. Is it invertible? If so, can you find a formula its inverse, f^{-1} ? If so, graph it on the same axes. Write down all horizontal and vertical asymptotes of $f(x)$ and $f^{-1}(x)$. Can you see a relationship between the two sets of asymptotes? Can you explain it?
8. Graph the function $f(x) = x^5 + x$. Is it invertible? If so, can you find a formula for the inverse? What are $f^{-1}(0)$, $f^{-1}(2)$, $f^{-1}(34)$, $f^{-1}(-2)$, and $f^{-1}(-34)$? Graph the inverse using these values.

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9. Is $f(x) = 2^x$ invertible? If so, write down some values of f^{-1} , then sketch its inverse. Does it have any asymptotes? Does $f(x)$ have any asymptotes? Be sure your answer line up with your answer in Question 7 above. Do not (yet) try to find a formula for f^{-1} .
10. Below are graphs of two functions with the line $y = x$ added to each (as a dashed line). By creating tables of values and plotting points, graph their inverses on the same set of axes. What do you conclude?



11. What you may have concluded is that the graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the $y = x$. Why must that always be true?

Summary

If f^{-1} is the inverse of f , then

1. $f(f^{-1}(x)) = x = f^{-1}(f(x))$.
2. $y = f^{-1}(x)$ if and only if $x = f(y)$.
3. Domain of $f =$ Range of f^{-1} and Domain of $f^{-1} =$ Range of f .
4. The graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y = x$.
5. f is invertible if and only if it is a one-to-one function, i.e. if $x \neq y$ then $f(x) \neq f(y)$, i.e. if f passes the “horizontal line test”.