Compound Interest

The purpose of this lab is to develop formulas for computing interest and to examine the effect on the return on an investment when interest rates and the number of compounding periods each year are varied. This will motivate the introduction of exponential functions and their inverses.

Part I: Introductory Exercises

Make sure you carefully show and explain how you arrived at your answers. For these problems you are *developing* a formula so you shouldn't be using a formula you maybe have previously learned!

1. Suppose you invest \$10 at 6% annual interest, compounded annually.
   (a) How much money would you have after one year?
   (b) How much money would you have after two years?
   (c) How much money would you have after ten years?
   (d) How much money would you have after t years?

2. Suppose instead you invest \$10 at 6% annual interest, compounded semi-annually (i.e., twice a year). That means that each half-year you add 3% to what you have accumulated.
   (a) How much money would you have after one year?
   (b) How much money would you have after two years?
   (c) How much money would you have after ten years?
   (d) How much money would you have after t years?

3. If we double the initial investment and make an investment of \$20, how much will we have after 10 years if the return is 6%, compounded annually ? after 100 years?

4. Now suppose we double the interest rate to 12%, compounded annually, but make only a \$10 initial investment? How much will we have after 10 years? after 100 years?

Part II: Pushing the Limit

5. Suppose we invest \$1 at 100% interest. How much will we have after one year if the interest is compounded

   (a) monthly?

   (b) daily?

   (c) every minute?

   For monthly compounding, n, the number of compounding periods per year is 12. For daily compounding $n = 365$ and for compounding every minute $n = 525600$. If we let $n \to \infty$, we say the investment is compounded continuously.

   (d) How much will we have after one year if the interest is compounded continuously? Experiment with your calculator using larger values of n.

   (e) As n increases, does the value of the investment increase? Does it increase without bound, that is, will the value of the investment eventually reach any amount as n increases?

Definition The number e is the amount of money you would have after one year of having \$1 invested at 100% interest, compounded continuously. That is:

$$e = \lim \underline{\hspace{3cm}}$$

6. Suppose we invest \$1 at an interest rate of 0.05 (i.e., 5%). How much will we have after one year if the interest is compounded continuously? Compare your result to $e^{0.05}$.

7. Suppose we invest \$1 at an interest rate of r. How much will we have after 1 year if the interest is compounded continuously? How much will we have after t years if the interest is compounded continuously?

Part III: Exercises

8. Which is a better investment: an investment at a 5.25 % annual rate compounded once a year or an investment at a 5.2 % continuous rate? Explain.

9. Suppose you invest, for one year, \$100 in a account with an interest rate of r compounded continuously. At the end of the year you would have $100e^r$ dollars. If the interest rate is doubled to $2r$ (and still compounded continuously), how much would you have to invest to get the same yield of $100e^r$ dollars?

10. You are about to invest in an account that pays 5% compounded continuously. How much would you have to invest now so that in 10 years you would have \$20,000?