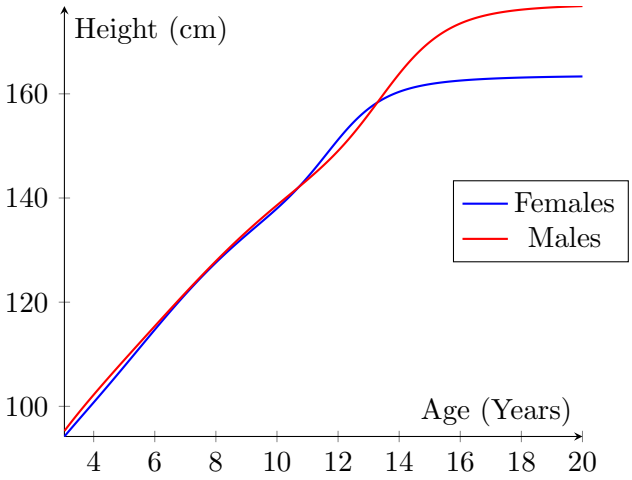
Human Growth

Purpose: In this lab, we will study patterns of growth for human children from age three to age twenty, as measured by the median height at these ages. We will compute *rates* of growth, and understand connections between the shape of the rate curve and the *concavity* of the height curve.

Background: Male and female children grow at different rates at various times of their lives. From the chart below, you can see that growth of the two sexes is similar for much of childhood, but that there are important differences. After reproducing this chart in a spreadsheet, we will explore these differences. We will then dive deeper after computing and charting the rates of growth.



Data Source

- Data Table of Stature-for-age Charts, Centers for Disease Control and Prevention (CDC). Retrieved from https://www.cdc.gov/growthcharts/html_charts/statage.htm.

Preliminaries

1. Open the spreadsheet, make a copy of it, and rename it with your group names.
2. It is good practice to examine the data and its source before analyzing it.
   (a) This is data for the *median* (or 50^{th} percentile) height of male and female children (in the sense of birth sex, not gender) *in the United States*.
      i. What does 'median height' of a child of a given sex and age mean?
      ii. Why might median heights of children differ in other countries? List at least three reasons.
   (b) In your own words, what does the number 95.27359 in cell B2 represent? Include units.

Part I: Charting the Data

3. Select columns A, B, and C. Then insert a chart. Select the 'Use Column A as Labels' and 'Use Row 1 as Headers' checkboxes. Then set the min and max on the y-axis to 90cm and 180cm respectively. Edit the title to be more succinct. Your resulting chart should look similar to the one above.

4. Use your chart and the data to make the following observations regarding median height male and female children. Write ages in *years*.

   (a) Males and female children are of similar height until approximately what age?

   (b) Female children are *slightly* taller between what ages?

   (c) After the age period in the previous part, _____ children continue to grow taller at a rapid rate, whereas _____ children have almost reached their adult height.

Part II: Rates of Growth

5. At 36.5 months, the median height of male children is 95.27359cm. At 37.5 months, it is 95.91475cm. What fraction of a year has passed in this time period? How much has the median height male child grown over this time period? Compute the *yearly* rate of growth of a median height male child between 36.5 and 37.5 months (Answer: 7.69392). Specify units.

6. In cell D3, insert a formula to compute this rate using the data in Columns A and B. Check that the number you get is the same as in the previous question. Then copy your formula all the way down the column. Lastly, repeat this in column E for female children. (Hint: you should get 0.1848 and 0.0696 in cells D206 and E206 respectively).

7. Select columns A, D, and E (hold down the Ctrl key on Windows, or Cmd on Mac to select non-contiguous columns). Then insert a chart. Select the 'Use Column A as Labels' and 'Use Row 1 as Headers' checkboxes.

8. Note that the rates of growth are always positive. Why does this make sense?

9. Use your chart of rates and the growth rate data to make the following observations regarding median height male and female children. Again, write ages in years.

   (a) Female children have _____ growth spurts, vs. only _____ for male children.

   (b) At what ages do each of these spurts peak? How fast are male and female children growing at the height of their last growth spurt?

   (c) What can you say about the graphs of *heights* from Part I *at* these points in time?

   (d) What happens to the rate of growth of male and female children *after* these peaks?

   (e) *After* these growth peaks, what happens to the graphs of *heights* from Part I?

Part III: Zooming In

In this part, we will focus on the heights of children between ages 8 and 14 years, in order to more deeply understand the shapes of the graphs and what those tell us.

10. Copy the data for these ages (from row 61 to row 134) to the second tab of the spreadsheet. Be sure to copy ages, as well as both heights and rates for both male and female children. Use 'Paste special → Paste values only' (or Ctrl/Cmd-Shift-V) to paste these in. If you've done this right, cell D2 of the second tab should have 5.94 in it.

11. We will now insert a chart showing all four columns (heights and rates) at the same time. We will display height on the left axis and rates on the right axis. This will allow us to plot the rates of growth on a different scale (and with different units) than the heights themselves. Follow the instructions below carefully, as this is a bit tricky:

    - Select all five columns and insert a chart.
    - Check the 'Use Column A as Labels' and 'Use Row 1 as Headers' checkboxes.
    - Under 'Customize', open the 'Vertical Axis' section and set the min and max to 125cm and 165cm respectively.
    - In the 'Series' section, select the 'Male rates of Growth' series, then change 'Axis' to 'Right Axis'. Then change the 'Line dash type' to dotted, and the 'Line color' to match the color of the height data line for male children. Repeat all this for female rates.
    - In the 'Chart and Axis Titles', select 'Vertical axis title', and enter the name of the quantity that appears on the left axis. Include units. Do the same for 'Right vertical axis title', and the horizontal axis title. Lastly, edit the chart title to be more succinct.
    - Make the chart large so that observations are easier to make.

    After you have completed this chart, please ask a TA to check it!

12. We will now compute the rates of change of the rates of change of height. You can think of these as the rates of *acceleration* of height, in a similar way to how we computed accelerations for a moving object as the rate of change of velocity, which is itself the rate of change of distance.

    (a) We computed rates of change as $\frac{\text{change in height}}{\text{change in time}}$ (see Question 5). To compute acceleration, we use $\frac{\text{change in rate}}{\text{change in time}}$. Compute the yearly rate of acceleration of a median height male child between 95.5 and 96.5 months (Answer: -0.4752). What are its units?

    (b) In columns F and G (starting in row 3), compute the acceleration of height for male and female children respectively. If you do this correctly, you should get -1.7424, and -1.7712 in cells F75 and G75 respectively. Insert titles for columns F and G.

13. We previously observed that female children have their maximal growth rate at approximately age 11.5 years (to be more precise, around age 140 months). After that point in time, their growth slows. After that point, you can see that the *rate* chart for female children is decreasing.

    (a) Given that the accelerations are the rate of change of the rate, what do you expect to be true about the acceleration of the height of female chidlren after 140 months? (Hint: compare Question 9.)

    (b) Fill in the following blanks: If the rates of change are decreasing, then the acceleration is _____. The graph of the *heights* is then *concave* _____. Check this is indeed the case on your graph.

14. We will now switch to considering the graphs for male children.

    (a) At age 8 years (96 months), is the height chart for male children concave up or down? This is hard to read off from the height chart itself. How can you use the rate chart and/or the accelerations to determine the concavity?

    (b) Until what age does the graph of height for male children have this concavity? What happens to the acceleration at that age?

    (c) After this point in time, the growth rate for male children increases.

    - What can you say about the values of the acceleration of their heights starting at this point?
    - Fill in the following blanks: If the rates of change are increasing, then the acceleration is _____. The graph of the *heights* is then *concave* _____. Check this is indeed the case on your graph.

    (d) We previously said that male children are growing at the fastest rates of their lives at age approximately 13.5 years. Use your acceleration data to find this point more precisely. We call points such as this and the one from Question 13 *inflection points* of the height graph. At an inflection point, concavity changes from up to down or vice versa.

15. The original graph (age 3-20) of height for male children has two inflection points, whereas the graph for female children has four. You already found some of these in in Questions 13 and 14. List all the IPs for each of the height curves. (Hint: how did we use the rate graph to find the IPs of the height graph in Question 13?)

16. (a) Explain why it makes sense that the graphs of height for both male and female children are concave down as they approach adulthood.

    (b) If $M(t)$ is the height of a median height male child at age t, what does the quantity $\lim_{t\to\infty} M(t)$ mean? Assume humans never lose bone mass (and therefore don't ever shrink)!

    (c) If $F(t)$ is the height of a median height female child, what is the approximate value of $\lim_{t\to\infty} M(t) - F(t)$? Convert the quantity to inches, and write a sentence with that value in it. Your answer should not include any mathematical concepts (e.g. limits), and should be easily understandable by people without math background.

Report

For your report, hand in complete answers to Questions 2, 8, 9, and 13-16. Your answers should be in complete sentences, and must all be fully justified. Any charts you refer to should be included, and any points or parts of the graphs referred to in your answers should be labeled or highlighted.