Exploring Limits

Note: In this lab, you may use a calculator (physical or online), a spreadsheet, or any other tool to complete the calculations. Parts of the lab will refer to online tools, and links will be provided. The answers to this lab should be written on paper or a Google Doc. If your group decides to use a Google Doc, be sure to include the pictures of all graphs you create!

Part I: Introduction to Limits

We will soon begin to explore the notion of a *derivative* of a function. To do so, we will first need to understand the concept of a *limit*. This lab will explore precisely that.

Rather than attempting to present a technical definition of a limit, we will plunge straight in and play with a few of them. The notion to keep in mind is:

Definition (vauge): The *limit* of a function $f(x)$ as x approaches some value a on the x-axis is the y-value we *expect* the function to take at that value of x, even if the actual value is different.

Example: If you know that the velocity of a car at 11: 59: 59 (one second before noon) was 44.9 miles per hour, at 11: 59: 59.9 (one tenth of a second before noon)44.99 miles per hour, at 12: 00: 0.1 (one tenth of a second after noon) 44.98 mph and at 12: 00: 01 (one second after noon) 44.95 mph, you expect that its velocity at 12 would have been _______ mph. If $v(t)$ is the velocity at time t (in seconds) after 12, fill in the following table:

t	-1	-0.1	0.1	1
$v(t)$				

1. Why isn't $t = 0$ included in the table?

2. What value appears to be the limit as t approaches 0 of $v(t)$? We write this:

$$\lim_{t\to 0} v(t) = \_\_\_\_\_\_\_.$$

Now to play with actual functions...

3. Approximate the limit $\displaystyle\lim_{x\to 0} \frac{(x-7)(x-2)}{x^2+1}$ by filling in the following table.

From the Right		From the Left	
x	$f(x)$	x	$f(x)$
0.1		-0.1	
0.01		-0.01	
0.001		-0.001	
0.0001		-0.0001	

4. Based on the entries in the table, what would you guess the limit of $f(x)$ is at 0?
5. Now calculate $f(0)$. How is this related to your answer to question 4?
6. What property of $f(x)$ explains this relationship?

Part II: Indeterminant Limits

There are times when things don't work out so nicely; guessing a limit simply by plugging in numbers is not always reliable.

Suppose that $f(x) = \dfrac{2x^2 - x - 1}{x - 1}$.

7. What value do you get when you try to calculate $f(1)$?

Indeterminate Values

Expressions that yield $\frac{0}{0}$ are referred to as *indeterminate values* and require more detailed analysis to determine their value (if their value even exists). They will come up often in Math 105L and 106L.

8. Fill in the table below to approximate $\lim_{x \to 1} \dfrac{2x^2 - x - 1}{x - 1}$.

From the Right		From the Left	
x	$f(x)$	x	$f(x)$
1.1		0.9	
1.01		0.99	
1.001		0.999	
1.0001		0.9999	

9. What does $\lim_{x \to 1} f(x)$ appear to be?
10. To see why this happens, factor the numerator of $f(x)$ and then simplify $f(x)$. Call this simplified expression $g(x)$. What is $g(1)$?
11. Note that unlike the function in Question 4, this function is not continuous. To see this, click on this link to Wolfram Alpha, and type in the following:

    `discontinuities of (2x^2-x-1)/(x-1)`

12. Is $f(1)$ defined?
13. The point $(1, 3)$ is called a "hole" in the graph of $f(x)$ because $f(x)$ seems to run through the point $(1, 3)$, yet it is not defined there. Copy the graph from Wolfram Alpha onto paper, including the hole. On a separate set of axes, draw the graph of $g(x)$ as well.

14. What is the relationship between $\lim_{x\to 1} f(x)$ and $g(1)$?

15. From your observations above, describe how to locate a 'hole' in a graph.

We will now consider the function $h(x) = \dfrac{x+10000}{9999.9999}$ and examine the limit $\lim_{x\to 0} h(x)$.

16. Fill out the table below to try and approximate this limit.

From the Right		From the Left	
x	$h(x)$	x	$h(x)$
0.1		-0.1	
0.01		-0.01	
0.001		-0.001	
0.0001		-0.0001	

17. Based on your table, what does $\lim_{x\to 0} h(x)$ appear to be?

18. Calculate $h(0)$.

19. What can you say about the weaknesses of the table method we've been using?

Part III: Limits at Infinity

20. We can also find limits "at infinity." To see what this means, graph the function

$$f(x) = \frac{2x^2+7}{x^2-x+1}$$

using Geogebra and zoom out a long way. Sketch and label this graph.

21. What does the graph do towards the edges of the screen? That is, What number does $f(x)$ seem to approach as x gets very large in the positive or negative direction?

These numbers are the limits "at infinity" and "at negative infinity" and is denoted by

$$\lim_{x\to +\infty} f(x) \text{ and } \lim_{x\to -\infty} f(x).$$

22. What does the value of this limit seem to be?

23. Describe the relationship between limits at infinity and horizontal asymptotes.

Part IV: Another Indeterminate Limit

Consider the function $f(x) = x^{\frac{1}{x-1}}$.

24. At what value of x is this function undefined?

25. Previously, we saw that the form $\frac{0}{0}$ is indeterminate. what is the form of this function at the discontinuity? There are a number of indeterminate forms, two of which $\frac{0}{0}$ and this one.

26. Use Wolfram Alpha to look at the discontinuity in this in the graph of this function like you did in Question 11. Copy the graph that appears.

27. If you type

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    limit of x^(1/(x-1)) as x approaches 1
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    into Wolfram Alpha, it will tell you what the limit is. What is the limit? Have you encountered this number before? It will come up a lot a bit later in the class!