Linear Models

Purpose: In this lab you will learn how to model a data set which appears to be linear, and use these ideas to model the velocity of a falling book.

Preview: How can you tell (visually and algebraically) if the data would be well-modeled by a linear function? If a linear function is appropriate, how can you find an equation for a linear function that will be a good model for the data? How good is the model you obtain? In the course of finding linear models you will encounter the idea of the average rate of change of a function.

Background: To look for a relationship between two varying quantities a common first step is to collect a set of data pairs for the two quantities. Once the data is collected, a scatter plot of the data pairs can be constructed. A scatter plot is simply a graph of points using the data pairs as coordinates. It is natural to ask at this point if there is an equation which explains the appearance of the scatter plot, i.e., an equation whose graph closely matches the scatter plot. This equation is called a mathematical model. Finding such a model allows us to predict the value of one quantity given the corresponding value of the other quantity. It can also yield new insights into the relationship between the quantities. You will find that many of the labs for this course are concerned with the modeling of data.

Part I: Height

Suppose you drop an object a short distance and measure its distance from the floor at various times during its fall. Will the object fall at a constant rate? Can the distance data be well-modeled by a linear function? Let's find out.

Below is a data set recorded from an experiment where a book was dropped from a height of about 1.542 meters. The table gives the height at every .03 second interval.

t	0	.060	.090	.120	.150	.180	.210	.240	.270	.300
s	1.542	1.512	1.475	1.451	1.403	1.381	1.291	1.215	1.083	.975
t	.330	.360	.390	.420	.450	.480	.510	.540	.570	.600
s	.884	.765	.621	.499	.550	.451	.469	.4275	0.00	.456

1. Open the spreadsheet from Sakai for this lab and make a copy, renaming it with your group names.

2. Make a scatter plot of this data (be sure to get the best window possible). Your plot should resemble half of a parabola – at least in part.

3. **Working with experimental data:** Do some points seem incorrect? What could explain the discrepancies? Edit your chart to exclude these points from it. Your plot should now more closely resemble half a parabola.

Part II: Motion and Rates of Change

4. **Examining the data:**
   (a) Calculate the *average rate of change* of the height over the interval from $t = 0$ seconds to $t = 0.06$ seconds. What are the units of the quantity you just calculated? What is the name of this quantity?
   (b) In Column C, insert formulas to calculate the average rates of change of the data. Be sure to only do this for the data that was not excluded from your chart above (the last cell in column C with data in it should now be $C14$).
   (c) What do you notice about your average rates of change? Do they trend upward or downward (in other words, overall, are they increasing or decreasing)? What does that mean in real world terms?
   (d) What characteristics of the graph indicate that the book was dropped rather than thrown downward

5. **Re-examining the average rates of change:**
   (a) What are the units for your average rates of change?
   (b) What do they represent in terms of the motion of the falling book?
   (c) Add another chart an additional chart to your spreadsheet for average rates of change vs. time[1].
   (d) From your scatter plot, what do you observe about the velocity of the falling book?

6. **Modeling the velocity:** Use the steps in Part I to find a linear model $v = mt + b$ for the velocity data, both by hand and by using the Linear Trend feature in your spreadsheet. Add both lines to your chart, and make sure you have formulas for both.

7. **Interpreting the model:**
   (a) What does the slope m represent in this case? (Hint: What are the units of the average rate of change of the average rates of change of the distances?)
   (b) What is the meaning of the fact that your data for v are negative?
   (c) What is the practical meaning of the fact that your model has negative slope?
   (d) Near the earth's surface the acceleration of gravity is approximately -9.8 meters/second per second. State at least two reasons why your calculation of the acceleration due to gravity differs from this.
   (e) According to your model, what was the velocity of the book at t = 0? Does this result agree with what happened during the experiment? If not, explain the discrepancy.
   (f) According to your model, how long would it take the book to reach a velocity of -50 meters per second ($\approx$ 110 miles per hour)? Does the book ever reach this velocity? Explain.

[1] To select two non-adjacent columns, select one, then hold down ctrl (or command on a Mac) and select the other.)

Linear Models Lab Report Form

Answer each of the following equations using one or two complete sentences.

1. What does the slope m represent in this case?
2. Explain why, in this situation, it makes sense that your data for v are negative.
3. What is the practical meaning of the fact that your model has a negative slope?
4. According to the linear trendline model, what was the velocity of the falling book at $t = 0$? In the experiment conducted that produced the data set you used in the lab, the people conducting the experiment reported that they dropped the book. What would this mean in terms of the velocity of the book at $t = 0$? How might you explain this discrepancy?
5. Assuming the height measurement is imperfect, explain why it is impossible to verify that the book was in fact dropped using measured height data. Specifically, if the second measured height is less than the first, explain why it is possible for the book to have been thrown upward, dropped, or thrown downward.