Relative Rates of Growth

> **Definition** g *dominates* f as $x \to \infty$ if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$

In this lab we explore which common functions dominate other common functions. L'Hôpital's rule gives us one way of checking dominance, but we will mostly use algebra.

First, note that if $\lim_{x\to\infty} \frac{f(x)}{g(x)} = 0$ and $\lim_{x\to\infty} \frac{h(x)}{f(x)} = 0$, then $\lim_{x\to\infty} \frac{h(x)}{g(x)} = 0$. That is, if g dominates f and f dominates h, then g dominates h.

Rank order the following functions in order of dominance (1 being the most dominant, 18 the least). Note that two of the functions are equivalent: $a(x)$ and $b(x)$ are equivalent if $a(x)$ does not dominate $b(x)$ and $b(x)$ does not dominate $a(x)$.

e^{x^2}	e^x	x^2	e^{2x}	e^{5x}	x^n $(n > 9)$
$x^{\frac{1}{10}}$	$\sqrt{x}$	xe^x	$\ln x$	$x \ln x$	x^x
$10x^9 + 20x^8$	e^{2x+1}	$5x$	5^x	$(\ln x)^2$	$x^2 \ln x$

e^{x^2}	___	e^x	___	x^2	___
e^{2x}	___	e^{5x}	___	x^n $(n > 9)$	___
$x^{\frac{1}{10}}$	___	$\sqrt{x}$	___	xe^x	___
$\ln x$	___	$x \ln x$	___	x^x	___
$10x^9 + 20x^8$	___	e^{2x+1}	___	$5x$	___
5^x	___	$(\ln x)^2$	___	$x^2 \ln x$	___