The Product Rule:

1. (a) The Product Rule states that:

\[ \frac{d}{dx} [u(x)v(x)] = \] 

(b) Thus, we can apply the Fundamental Theorem of Calculus and obtain

\[ u(x)v(x) = \int \quad + \int \quad . \]

(c) We can now write \( du = u'(x) \, dx \) and \( dv = v'(x) \, dx \). Rearranging the terms, we get the

**Integration by Parts Formula:**

In order for this to be useful, the integral on the right needs to be easier than the integral on the left. How do we pick \( u \) and \( dv \)? Well, to start with, \( dv \) should to be easy to integrate!

**Examples:**

2. \( \int xe^x \, dx \)
   
   \[
   \begin{align*}
   u &= \\
   dv &= \\
   du &= \\
   v &= 
   \end{align*}
   \]

3. \( \int_1^2 \ln x \, dx \)
   
   \[
   \begin{align*}
   u &= \\
   dv &= \\
   du &= \\
   v &= 
   \end{align*}
   \]

4. \( \int x \ln x \, dx \)
5. $\int_0^\pi x \sin x \, dx$

6. $\int_0^1 x \sqrt{x + 1} \, dx$ (Compare to worksheet 7-2, Q15. Which method do you prefer?)

7. $\int_1^2 x^2 e^x \, dx$ (Hint: Integrate by parts twice)

8. $\int \arctan x \, dx$ (Hint: Let $u = \arctan x$.)
9. \[ \int e^x \cos x \, dx \] (Hint: Integrate by parts twice, and carefully observe the equation you obtain.)

10. \[ \int x(\ln x)^3 \, dx \] (Hint: Integrate by parts times.)

11. \[ \int xe^{-x^2} \, dx \] (Hint: Don’t work too hard!)