The Product Rule:

1. (a) The Product Rule states that:

\[ \frac{d}{dx} [u(x)v(x)] = \text{________________________} \]

(b) Thus, we can apply the Fundamental Theorem of Calculus and obtain

\[ u(x)v(x) = \int + \int . \]

(c) We can now write \( du = u'(x) \, dx \) and \( dv = v'(x) \, dx \). Rearranging the terms, we get the

Integration by Parts Formula:

In order for this to be useful, the integral on the right needs to be easier than the integral on the left. How do we pick \( u \) and \( dv \)? Well, to start with, \( dv \) should to be easy to integrate!

Examples:

2. \[ \int xe^x \, dx \]
   \[ u = \quad dv = \]
   \[ du = \quad v = \]

3. \[ \int_1^2 \ln x \, dx \]
   \[ u = \quad dv = \]
   \[ du = \quad v = \]

4. \[ \int x \ln x \, dx \]
5. \[ \int_{0}^{\pi} x \sin x \, dx \]

6. \[ \int_{0}^{1} x \sqrt{x + 1} \, dx \] (Compare to worksheet 7-2, Q15. Which method do you prefer?)

7. \[ \int_{1}^{2} x^2 e^x \, dx \] (Hint: Integrate by parts twice)

8. \[ \int \arctan x \, dx \] (Hint: Let \( u = \arctan x \).)
9. \[ \int e^x \cos x \, dx \] (Hint: Integrate by parts twice, and carefully observe the equation you obtain.)

10. \[ \int x (\ln x)^3 \, dx \] (Hint: Integrate by parts times.)

11. \[ \int x e^{-x^2} \, dx \] (Hint: Don’t work too hard!)