Review

1. (a) The Chain Rule states that:
   \[ \frac{d}{dx}[f(g(x))] = \]
   (b) By taking the antiderivatives of both sides, we get:
   \[ \int f'(g(x))g'(x) \, dx = \]

Method of \textbf{u}-substitution

2. (a) Suppose we want to evaluate \( \int f'(g(x))g'(x) \, dx \). Let \( u = g(x) \).
   Then \( du = \) \( dx \).
   (b) Hence,
   \[ \int f'(g(x))g'(x) \, dx = \]
   \[ = \]
   \[ = \]
   (c) Note also that
   \[ \int_a^b f'(g(x))g'(x) \, dx = \]
   \[ = \]

Examples

Note: Not all of these require substitutions. One of the most important integration skills is spotting what is the easiest method to use for a given integral.

3. \( \int x(1 + x^2)^5 \, dx \)
4. \( \int_0^{\frac{\pi}{2}} \sin x \cos x \, dx \)

5. \( \int_0^1 \frac{x}{\sqrt{x^2+1}} \, dx \)

6. \( \int \frac{1}{3x+1} \, dx \)

7. \( \int \frac{1}{e^{3x}} \, dx \)
8. \( \int \sin^2 x \, dx \) (Hint: \( \cos 2x = 1 - 2\sin^2 x \))

9. \( \int_0^2 \frac{e^x}{1 + e^{2x}} \, dx \) (Hint: What’s another way to write \( e^{2x} \)? Rules of exponentials...)

10. \( \int \frac{\ln x}{x} \, dx \)

11. \( \int \cos^2 x \sin x \, dx \)
12. \( \int xe^{-x^2} \, dx \)

13. \( \int \tan x \, dx \) (Hint: write \( \tan x \) as a quotient of two other trig functions.)

14. \( \int \frac{2x}{\sqrt{1 - x^4}} \, dx \)

15. \( \int_0^1 x\sqrt{x+1} \, dx \) (Hint: Let \( u = x + 1 \). Then \( x = \ldots? \))