Review

1. (a) The Chain Rule states that:
\[ \frac{d}{dx}[f(g(x))] = \]

(b) By taking the antiderivatives of both sides, we get:
\[ \int f'(g(x))g'(x) \, dx = \]

Method of $u$-substitution

2. (a) Suppose we want to evaluate \( \int f'(g(x))g'(x) \, dx \). Let \( u = g(x) \).

Then \( du = \ _____ \, dx \).

(b) Hence,
\[ \int f'(g(x))g'(x) \, dx = \]
\[ = \]

(c) Note also that
\[ \int_a^b f'(g(x))g'(x) \, dx = \]
\[ = \]

Examples

Note: Not all of these require substitutions. One of the most important integration skills is spotting what is the easiest method to use for a given integral.

3. \( \int x(1 + x^2)^5 \, dx \)
4. \[ \int_{0}^{\pi/2} \sin x \cos x \, dx \]

5. \[ \int_{0}^{1} \frac{x}{\sqrt{x^2 + 1}} \, dx \]

6. \[ \int \frac{1}{3x + 1} \, dx \]

7. \[ \int \frac{1}{e^{3x}} \, dx \]
8. \( \int \sin^2 x \, dx \) (Hint: \( \cos 2x = 1 - 2 \sin^2 x \))

9. \( \int_0^2 \frac{e^x}{1 + e^{2x}} \, dx \) (Hint: What’s another way to write \( e^{2x} \)? Rules of exponentials...)

10. \( \int \frac{\ln x}{x} \, dx \)

11. \( \int \cos^2 x \sin x \, dx \)
12. $\int x e^{-x^2} \, dx$

13. $\int \tan x \, dx$ (Hint: write $\tan x$ as a quotient of two other trig functions.)

14. $\int \frac{2x}{\sqrt{1-x^4}} \, dx$

15. $\int_0^1 x\sqrt{x+1} \, dx$ (Hint: Let $u = x + 1$. Then $x = \ldots$?)