Review

1. • If $f(x)$ is decreasing on $[a, b]$, then using a left-hand sum to estimate the area ‘under’ the graph of $f(x)$ between $x = a$ and $x = b$ gives an ________ of the true area, and using a right-hand sum gives an ________ of the true area.

• If $f(x)$ is increasing on $[a, b]$, then using a left-hand sum to estimate the area ‘under’ the graph of $f(x)$ between $x = a$ and $x = b$ gives an ________ of the true area, and using a right-hand sum gives an ________ of the true area.

• Draw examples to show why the above are true.

2. • To estimate the area, we divide $[a, b]$ into $n$ equal subintervals of width $\Delta x = _____$.

• In a left-hand sum, the area of the $i^{th}$ rectangle is ________.

• In a right-hand sum, the area of the $i^{th}$ rectangle is ________.

3. Consider the function $f(x) = (x - 1)(x - 2)$. Draw two copies of a graph of this function, then draw a left-hand sum and a right-hand sum with $n = 3$ subintervals for the area ‘under’ the graph between $x = 0$ and $x = 1.5$. Compute both sums.

4. Why did I put ‘under’ in quotes?
The Midpoint and Trapezoid Sums

Definition In a **midpoint sum**, we choose to compute the height of each rectangle in the *center* of each subinterval instead of on the left or right.

5. Fill in the blanks:
   - When we divide the interval \([a, b]\) into \(n\) equal subintervals, the left hand of the \(i^{\text{th}}\) subinterval is _______, and its right hand is _______.
   - So the center of the \(i^{\text{th}}\) subinterval is _______.
   - The area of the \(i^{\text{th}}\) rectangle in a midpoint sum is therefore _________.

Definition The **trapezoid sum** is the average of the left hand and right hand sums.

6. Draw and then calculate \((LHS)_2\), \((RHS)_2\), \((TRAP)_2\) and \((MPS)_2\) for the following areas:
   - (a) The area ‘under’ \(x^2 - 1\) between \(x = 0\) and \(x = 4\).

\[
(LHS)_2 = _____ \quad (RHS)_2 = _____ \quad (TRAP)_2 = _____ \quad (MPS)_2 = _____
\]

   - (b) The area ‘under’ \(\sqrt{x} - 2\) between \(x = 0\) and \(x = 4\).

\[
(LHS)_2 \approx _____ \quad (RHS)_2 \approx _____ \quad (TRAP)_2 \approx _____ \quad (MPS)_2 \approx _____
\]
(c) Assuming one of $TRAP_2$ and $MPS_2$ is an overestimate of the true area, and the other an underestimate, which one is which in each of the two cases above?

(d) When does it seem the MPS and the average of the LHS and RHS are over/underestimates of the true area?

7. Let’s demonstrate why your answer to the last question is true. To do this, we must recast these two sums as sum of areas of trapezoids:
Conclusion

8. • If \( f(x) \) is \( \underline{\text{increasing}} \), then \( LHS_n \) is an underestimate of the true ‘area’ under \( f(x) \) and \( RHS_n \) is an overestimate for it.

• If \( f(x) \) is \( \underline{\text{decreasing}} \), then \( LHS_n \) is an overestimate of the true ‘area’ under \( f(x) \) and \( RHS_n \) is an underestimate for it.

• If \( f(x) \) is \( \underline{\text{non-increasing}} \), then \( MPS_n \) is an underestimate of the true ‘area’ under \( f(x) \) and \( \text{TRAP} = \frac{LHS_n + RHS_n}{2} \) is an overestimate for it.

• If \( f(x) \) is \( \underline{\text{non-decreasing}} \), then \( MPS_n \) is an overestimate of the true ‘area’ under \( f(x) \) and \( \text{TRAP} = \frac{LHS_n + RHS_n}{2} \) is an underestimate for it.