## Reminder: Inverse Functions

Defintion A function $f$ is invertible if the following equivalent conditions are fulfilled:

- $f$ is a one-to-one function;
- $x \neq y \Rightarrow f(x) \neq f(y)$;
- $f(x)$ passes the horizontal line test.

If $f^{-1}(x)$ is the inverse of $f$, then

- $f\left(f^{-1}(x)\right)=x$ for all $x$ in the domain of $f^{-1}$, and $f^{-1}(f(x))=x$ for all $x$ in the domain of $f$.
- $y=f^{-1}(x) \Leftrightarrow x=f(y)$.
- $\operatorname{Domain}\left(f^{-1}\right)=\operatorname{Range}(f)$ and $\operatorname{Range}\left(f^{-1}\right)=\operatorname{Domain}(f)$.
- The graphs of $f(x)$ and $f^{-1}(x)$ are symmetric about the line $y=x$.


## Inverting Trigonometric Functions

Now let's apply what we know about inverse functions in general to the specific functions $y=\sin x$ and $y=\tan x$.
$\sin x$

1. (a) On the axes below graph $y=\sin x$ on $[-2 \pi, 2 \pi]$.

(b) Why is $\sin x$ not invertible on $[-2 \pi, 2 \pi]$ ?
(c) What is the simplest domain on which $\sin x$ is invertible?
(d) Let's call the inverse of $\sin x$ on that domain $\sin ^{-1} x$, or $\arcsin x$. What is the domain of $\sin ^{-1} x$ ? The range?
2. (a) What is $\sin \left(\sin ^{-1} x\right)$ ? For which values of $x$ is that true?
(b) What is $\sin ^{-1}(\sin x)$ ? For which values of $x$ is that true?
(c) If $y=\sin ^{-1} x$, then $x=$ $\qquad$ .
(d) The axes below show a graph of $\sin x$ on the domain $\left.\left[-\frac{\pi}{2},-\frac{\pi}{2}\right]\right]$. Sketch a graph of $y=\sin ^{-1} x$. The line $y=x$ is shown dotted to help you.


## $\cos x$

3. (a) On the axes below graph $y=\cos x$ on $[-2 \pi, 2 \pi]$.

(b) Why is $\cos x$ not invertible on $[-2 \pi, 2 \pi]$ ?
(c) What is the simplest domain on which $\cos x$ is invertible?
(d) Let's call the inverse of $\cos x$ on that domain $\cos ^{-1} x$, or $\arccos x$. What is the domain of $\cos ^{-1} x$ ? The range?
4. (a) What is $\cos \left(\cos ^{-1} x\right)$ ? For which values of $x$ is that true?
(b) What is $\cos ^{-1}(\cos x)$ ? For which values of $x$ is that true?
(c) If $y=\cos ^{-1} x$, then $x=$ $\qquad$
(d) The axes below show a graph of $\cos x$ on the domain $[0, \pi]$. Sketch a graph of $y=\cos ^{-1} x$. The line $y=x$ is shown dotted to help you.


## $\tan x$

5. (a) On the axes below graph $y=\tan x$ on $[-2 \pi, 2 \pi]$.

(b) Why is $\tan x$ not invertible on $[-2 \pi, 2 \pi]$ ?
(c) What is the simplest domain on which $\tan x$ is invertible?
(d) Let's call the inverse of $\tan x$ on that domain $\tan ^{-1} x$, or $\arctan x$. What is the domain of $\tan ^{-1} x$ ? The range?
6. (a) What is $\tan \left(\tan ^{-1} x\right)$ ? For which values of $x$ is that true?
(b) What is $\tan ^{-1}(\tan x)$ ? For which values of $x$ is that true?
(c) If $y=\tan ^{-1} x$, then $x=$ $\qquad$ .
(d) The axes below show a graph of $\tan x$ on the domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$. Sketch a graph of $y=\tan ^{-1} x$. The line $y=x$ is shown dotted to help you. Be sure to indicate any asymptotes!


## Homework Questions

1. Evaluate the following. Be sure to express your answers in radians.
(a) $\sin ^{-1}(0)(b) \sin ^{-1}(1)(c) \sin ^{-1}(-1)(d) \cos ^{-1}(0)(e) \cos ^{-1}(1)$
(f) $\cos ^{-1}(-1)(\mathrm{g}) \tan ^{-1}(0)(\mathrm{h}) \tan ^{-1}(1)$ (i) $\tan ^{-1}(-1)$
2. In which quadrants do each of the following statements hold?
(a) $\sin \theta>0$ and $\cos \theta<0$ (b) $\tan \theta>0$ (c) $\tan \theta<0$
(d) $\sin \theta<0$ and $\cos \theta>0$ (e) $\cos \theta<0$ and $\tan \theta>0$
3. For each of the following, solve the equation for the given range. Be sure to express your answers in radians.
(a) $\cos (\theta)=\frac{1}{\sqrt{2}}$ for $0 \leq \theta \leq 4 \pi$ (b) $\sin (\theta)=\frac{\sqrt{3}}{2}$ for $-\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$
(c) $\tan (\theta)=-\frac{1}{\sqrt{3}}$ for $-2 \pi \leq \theta \leq 0$ (d) $\cos (\theta)=0.42$ for $0 \leq \theta \leq 2 \pi$
(e) $\sin (\theta)=0.91$ for $-2 \pi \leq \theta \leq 2 \pi$ (f) $\tan (\theta)=2.14$ for $0 \leq \theta \leq 5 \pi$
4. For each of the following, solve for all $x$ with $0 \leq x \leq 2 \pi$. Express all your answers in radians and give exact answers if possible.
(a) $2 \cos x=1$ (b) $\tan x=\sqrt{3}-2 \tan x$ (c) $3 \sin ^{2} x+4=5$
(d) $4 \tan x+3=2$ (e) $3 \cos ^{2} x+2=3-2 \cos x$ (f) $3 \sin ^{2} x+3 \sin x+4=3-2 \sin x$
